Pricing and Interconnection Agreements in Network Markets

Inaugural-Dissertation

zur Erlangung des Grades akademischen Grades eines Doktors

der Wirtschafts- und Sozialwissenschaften

der Wirtschafts- und Sozialwissenschaftlichen Fakultät

der Christian-Albrechts Universität zu Kiel

vorgelegt von

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aus München

Berlin, 2008

Fluffy's Buch

Acknowledgement

Two people have been of invaluable help to me in preparing this thesis. First, Toker Doganoglu, who co-authored chapters 3 and 4 introduced me to interconnection pricing and taught me a lot about critical thinking. Second, Till Requate, who read and re-read all chapters very carefully and showed me "the way out".

I want to thank my colleagues and friends at the Institute for Financial Econometrics at the Universities of Kiel and Munich and the Center for Information and Network Economics Daniel Cerquera, Lukasz Gryzbowski, Markus Haas, Christoph Hartz, Christian Pigorsch, Corinna Prange and Katharina Sailer.

Financial aid of the Volkswagen Stiftung during 2002 and 2005 is gratefully acknowledged.

Last but not least, I want to thank Anke for everything — being my wife, my friend, my critic and for listening.

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Chapter 1

Introduction

Ever since the beginning of the nineties, interconnection pricing emerged as a high priority regulatory policy question. As more and more state-owned monopolies were privatized, their essential facility (or simply their network) had to be opened to competition. The most viable practice was call-by- call services in telecommunications markets. Entrants were allowed to buy capacity of the incumbent's network in order to provide service to customers. Because the incumbent always has an incentive to foreclose the market, these agreements between incumbent and entrant were and still are in the focus of regulatory authorities all over the world.

Although this so-called one-way interconnection model is still valid, competition between fully fledged networks has become of growing importance. Today cable providers are also capable of offering telecommunication services. However they dispose of a mature network and can bypass the former incumbent's essential facility. Early models of network competition, such as Laffont, Rey, and Tirole (1998a) and Armstrong (1998) highlight the collusive role of interconnection charges in these settings. Hence the regulator is called for.

The present thesis looks at instruments that can be used to regulate interconnection markets. It is comprised of three papers on access pricing in one-way as well as in two-way interconnection models. Chapter 2 looks at instruments that can be employed by the regulator, chapters 3 and 4 introduce fully decentralized mechanisms, where the role of the regulator is reduced to a minimum.

Chapter 2 introduces a one-way interconnection model with a vertically integrated incumbent and an entrant who uses the incumbent network. Firms produce differentiated services and compete for customers. Traditional literature on interconnection pricing uses primarily per-unit access charges, hence the interconnection payment depends on the overall amount of traffic exchanged. In this paper we present a different approach to pricing interconnection. Because products are differentiated, the demand for end service is directly dependent on both firms' retail prices. We generalize the access payment function and make it solely dependent on market prices. I show that a per-unit access price is a special case of a contract on both firms' retail prices. Because such a function uses at least two instruments, we are able to reproduce specific access charge rules. Furthermore, due to the increased number of instruments, the set of possible outcomes is increased as compared to a per-unit access charge.

By comparing different benchmark cases and computing their respective parameters, we show that they can be used as a preliminary measure of goodness of the access mechanism. Additionally, we show that bill & keep, that is providing access free of charge, yields lower retail prices than regulating a per-unit access charge at cost.

Chapter 3 uses the findings of chapter 2 and introduces as particular mechanism based on firms' retail prices. We use the exact same model of industry structure, i.e. a vertically integrated incumbent and an entrant, that uses network capacity as an essential input. Product market competition is imperfect, i.e. products are differentiated.

The access pricing mechanism is fully decentralized, i.e. other than setting up the rules of the game, the regulator does not intervene in the market at all. This is a particular strength of our mechanism, since traditional access pricing rules always require the regulator to dispose of almost full market information[1]. The access payment mechanism is modeled as a game between the incumbent and the entrant. Each firm determines a parameter in a linear payment function on firms' downstream retail prices. We show that this results in lower prices and higher welfare than the regulated per-unit access charge approach. Furthermore we present conditions for which the equilibrium in the model

[1]Of course, asymmetric information models have been employed to regulatory problems, the classical paper is certainly Baron and Myerson (1982). Vickers (1995) applies their analysis to telecommunications industries. In chapter 3 the regulator has no information whatsoever, which is an even stronger assumption

coincides with the optimal Ramsey outcome.

Chapter 4 draws on the literature pioneered by Laffont, Rey, and Tirole (1998a), Laffont, Rey, and Tirole (1998b) and Armstrong (1998). It uses a two-way interconnection model, with network-based price discrimination, i.e. firms charge different prices for their own and the competitor's network. The paper of Gans and King (2001) shows, that when firms are able to charge two-part tariffs, a per-unit access charge is used as a collusive device. The optimal equilibrium access charge is below marginal cost, hence customers save on off-net calls. However this is more than offset by an increase in the flat payment and overall welfare is reduced as compared to a regime with access priced at marginal cost

Using a consumer's net-utility function we derive a new retail price mechanism. Instead of competing in flat fees, firms offer net-utility levels. In that case, the monthly payment is derived from the equilibrium net-utility level and the gross utility of making calls to both networks. The paper shows that using such a mechanism eliminates the collusive power of access charge is vanished and overall welfare is increased.

Since net-utility is a function of the flat payment, the result is reminiscent of the literature on Bertrand and Cournot Competition with substitutes. We employ a graphical argument used in Cheng (1985) to show that competition in net-utility levels is a dominated strategy. The same is true for price competition when prices are complements.

Chapter 2

Evaluating Interconnection Agreements

2.1 Introduction

Interconnection pricing emerged as one of single most important problems after the wave of regulation started swamping network industries in the early nineties.

In a nutshell, the theory (and practice) of interconnection pricing[1] is concerned with finding the right compensation for using another agent's essential facility. For network industries such as telecommunications, energy, railroad or postal services this is a physical network. In order to provide service to customers, firms have to be granted access to the owner's network. The rates and terms at which access is allowed is the access/interconnection price.

Despite the importance of access pricing in practice, the only contractual form that has been studied intensively is the per-unit access price[2][3]. The literature on interconnection pricing takes this per-unit contract as given and derives optimal access pricing rules, most notably the Efficient Component Pricing Rule (ECPR) and optimal Ramsey prices.

[1]Throughout the paper we use both access and interconnection pricing equally.

[2]For extensive surveys on interconnection theory and policy, the reader is referred to Laffont and Tirole (2000) and Armstrong (2002b)

[3]A notable exemption is Jeon and Hurkens (forthcoming). He studies access pricing contracts that are not only conditioned on output but also on prices.

There are two reasons why this approach is questionable. First of all, a per-unit access contract is only one special scheme of many possibilities. It seems unreasonable to restrict one's options in such a way that market outcomes are excluded ex ante. Secondly optimal per-unit access pricing requires information that may not be easily to acquire in reality. Network industries are generally governed by large amounts of fixed (sunk) cost[4] and very small marginal cost. Hence charging per-unit prices might not be cost effective after all, because on the one hand, there might be no cost associated to the unit, prices are based on. On the other hand, cost accounting procedures and deriving cost estimates is costly in itself. Major telecommunications companies spend a non-negligible fraction of budget to compute estimates of marginal cost whose sole purpose is reporting to regulatory authorities.

In the paper we want to examine these two points in particular. We use a simple model of one-way interconnection with an incumbent who is the owner of a network and an entrant who acquires network service in order to produce a service. Both firms compete in prices by offering differentiated products.

Given the model framework we compute access pricing formulas, that are proposed in the literature. Using a simple access payment function, that is based on both customer prices, we show that it is possible to reproduce these results. Furthermore this interconnection payment is able to implement more market outcomes than regulating a per-unit access charge because there are more instruments available. Additionally it gives a simple indication, which access payment is socially preferable, without using any additional information.

Finally the paper opts in favour of fixed fee compensation as compared to access price regulation. It is shown show that under the assumption of substitutable services, marginal (access) prices of zero are welfare improving. This also takes account of the inherent cost structure of the industries with large fixed cost and negligible marginal cost.

The paper is organized as follows. Section 4.2 introduces the general model proofs the existence and uniqueness of equilibrium and computes benchmarks. In section 2.3 we analyze different access pricing schemes and derive the main results of the paper, section 4.5 concludes.

[4]The impact of sunk costs is discussed in Hausman (1997).

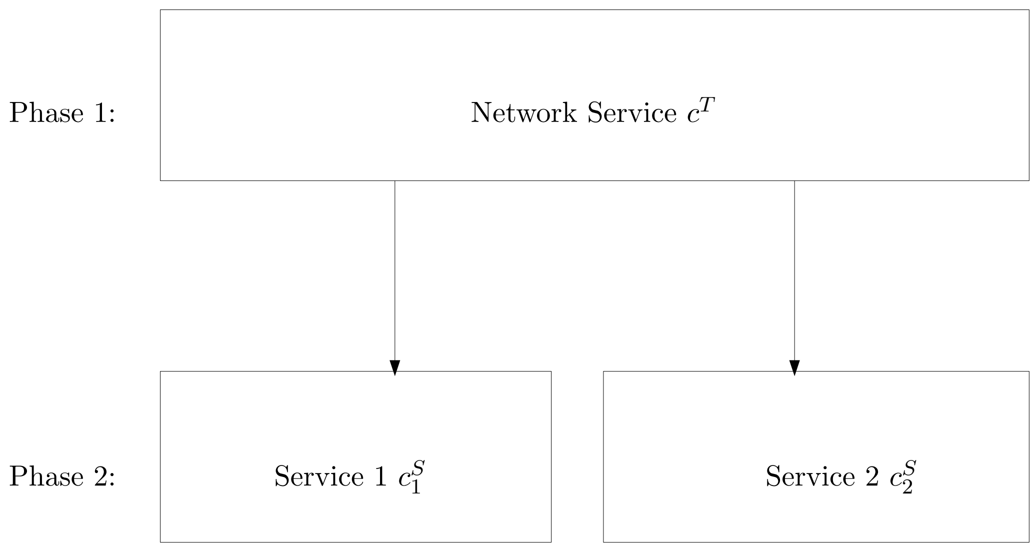

Figure 2.1: Production Structure

2.2 A Stylized Model

2.2.1 Multiproduct Monopoly

Consider a monopolist M who produces to two differentiated services, service 1 and 2. Production of either service requires two inputs. The first one is the essential facility, in our case a network which is owned by M. For each unit of end service, M incurs a marginal cost of c^T (see figure 2.1).

The second component is service specific, i.e. for each unit of service 1, the firm incurs a cost of c_1^S and for providing one unit of service 2, c_2^S is incurred. For the reminder of the paper we assume that $c_1^S = c_2^S = c^S$ The production flow is depicted in figure 2.1. We assume that in order to produce service $i \in \{1, 2\}$, the monopolist has to incur a service specific fixed cost F_i.

This is a simple model of a telecommunications industry with one firm supplying both services, voice and data. Network service, i.e. provision, maintenance and operation of the physical network is the essential input. Service 1 and 2 may be interpreted as voice and data applications.

Products are imperfect substitutes, hence the monopolist faces differentiated demands for both services. Demand for end service i is given by

$$q_i = q_i(p_i, p_j) \quad \forall\, i \in \{1,2\} \;\; i \neq j$$

with derivatives satisfying

$$\frac{\partial q_i(p_i,p_j)}{\partial p_i} < 0 \quad \frac{\partial^2 q_i(p_I,p_j)}{\partial {p_I}^2} \geq 0 \quad \frac{\partial^2 q_i(p_I,p_j)}{\partial p_I \partial p_j} \geq 0 \quad \forall\, i \in \{I,E\} \;\; i \neq j.$$

Having defined the production process as well as the demand side, M's profit function is given by

$$\Pi^M = (p_1 - c^T - c^S)q_1(p_1,p_2) - F_1 + (p_1 - c^T - c^S)q_2(p_1,p_2) - F_2$$

M maximizes his profit by finding optimal retail prices. These prices have to satisfy the first-order conditions[5]

$$(p_1^M - c^T - c^S)\frac{\partial q_1(p_1^M,p_2^M)}{\partial p_1^M} + q_1(p_1^M,p_2^M) + (p_2^M - c^T - c^S)\frac{\partial q_2(p_1^M,p_2^M)}{\partial p_1^M} = 0 \quad (2.1)$$

$$(p_2^M - c^T - c^S)\frac{\partial q_2(p_1^M,p_2^M)}{\partial p_2^M} + q_2(p_1^M,p_2^M) + (p_1^M - c^T - c^S)\frac{\partial q_1(p_1^M,p_2^M)}{\partial p_2^M} = 0 \quad (2.2)$$

For the market to be viable, $\Pi^M \geq 0$ i.e.

$$(p_1^M - c^T - c^S)q_1(p_1^M,p_2^M) + (p_2^M - c^T - c^S)q_2(p_1^M,p_2^M) \geq F_1 + F_2$$

.

In equilibrium both services are offered at a price above marginal cost $c^T + c^S$ and both services are bought. Let us now introduce the interconnection model, where network service is supplied as an input to an entrant and the incumbent, i.e. the former monopolist competes in the market for end services.

2.2.2 One-way Interconnection

Suppose that service 2 is provided by another firm, the entrant[6] E. In order to do so, he has to acquire network services from the incumbent I, who is proprietary of a physical network. In return, E has to make an access payment $A(p_I, p_E)$. The industry is depicted in figure 2.2.

[5] We assume that the second order conditions are satisfied, hence (2.1) and (2.2) characterize a maximum.

[6] From here on, all variables related to E are indexed with an E instead of a 2. Subsequently, all incumbent's variables are indexed with an I.

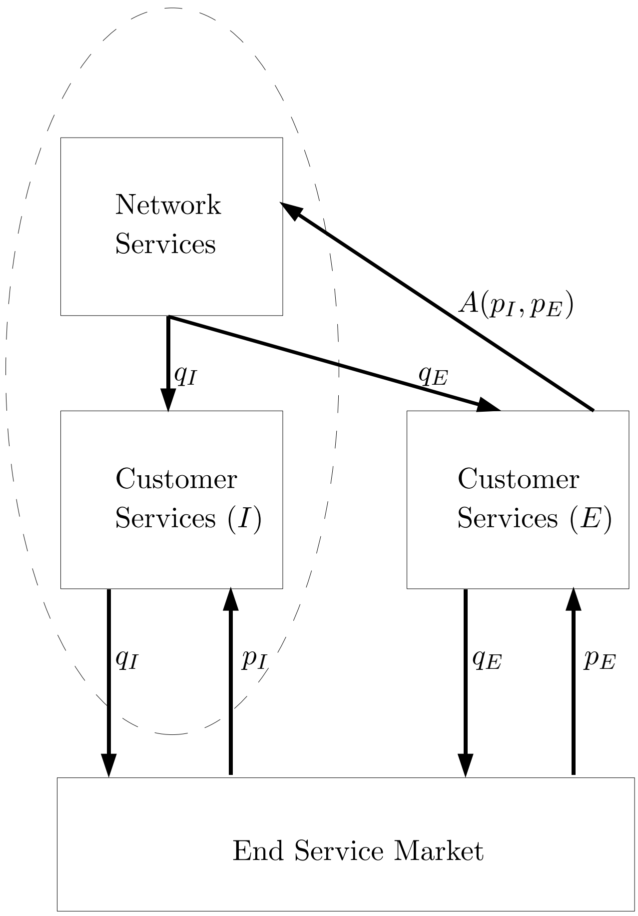

Figure 2.2: Up and Downstream Market

Providing a unit of either I or E's service requires a unit of network service and one unit of service specific input. Firm I provides a unit of capacity at marginal cost of c_T. The service specific input is provided by each firm at marginal cost of c^S. The access payment $A(p_I, p_E)$ is an arbitrary function of both retail prices. We implicitly assume that the cost of setting up a new fully fledged network is prohibitively high and bypass is not possible in any way. Firm I is a natural monopolist in the upstream market.

The model is build with telecommunications industry in mind. Network service is the transmission of data, which is provided by the network owner. Individual services such as voice services, email, P2P and the like are provided by independent firms who employ this network. They compete with the network proprietary who also offers a differentiated service using his network[7].

Equipped with the building blocks of the model, we can write both firms' profit functions as

$$\Pi^I = (p_I - c^T - c^S) q_I(p_I, p_E) - c^T q_E(p_I, p_E) + A(p_I, p_E) - F_I \tag{2.3}$$
$$\Pi^E = (p_E - c^S) q_E(p_I, p_E) - A(p_I, p_E) - F_E \tag{2.4}$$

The model is setup as a two-stage game. In stage one, the access mechanism is determined. This involves the structure and the level of the payment. In stage two, firms set their prices simultaneously given $A(p_I, p_E)$. Note that this is a game of full information. Hence the first-order conditions in the second stage of the game are given by

$$(p_I - c^T - c^S)\frac{\partial q_I(p_I, p_E)}{\partial p_I} + q_I(p_I, p_E) - c^T \frac{\partial q_E(p_I, p_E)}{\partial p_I} + \frac{\partial A(p_I, p_E)}{\partial p_I} = 0 \tag{2.5}$$
$$(p_E - c^S)\frac{\partial q_E(p_I, p_E)}{\partial p_E} + q_E(p_I, p_E) - \frac{\partial A(p_I, p_E)}{\partial p_E} = 0 \tag{2.6}$$

Almost all contributions on access or interconnection pricing make a very specific functional assumption on $A(p_I, p_E)$. They use linear per-unit access prices[8], so the access payment is proportional to the entrants demand. Notice that by assuming a differentiated goods demand, using a per unit access charge a is a special case of our arbitrary function $A(p_I, p_E)$ since

$$A(p_I, p_E) = a\ q_E(p_I, p_E).$$

The functional form imposed on $A(p_I, p_E)$ is determined by the demand function. The level of a is either set by the incumbent or regulated in some form.

2.2.3 Existence and Uniqueness

Let us begin the formal analysis by stating the the following proposition:

[7] Although the adjacency to telecommunications industries is apparent, the model can also be employed to traditional industries. We may think of a producer of tower cranes who rents his cranes to construction firms and at the same time sells his products to competitors who also rent cranes.

[8] An exception is Jeon and Hurkens (forthcoming). He uses an arbitrary mechanism that, in equilibrium, depends on the structure of marginal cost.

Proposition 2.1. *In the game described by (2.3) and (2.4) exists an equilibrium price pair p_I^* and p_E^* whenever*

$$c^T \frac{\partial^2 q_E(p_I,p_E)}{\partial p_I \partial p_E} - (p_I - c^T - c^S)\frac{\partial^2 q_I(p_I,p_E)}{\partial p_I \partial p_E} - \frac{\partial q_I(p_I,p_E)}{\partial p_E} \leq \frac{\partial^2 A(p_I,p_E)}{\partial p_I \partial p_E} \quad (2.7)$$

$$(p_E - c^S)\frac{\partial^2 q_E(p_I,p_E)}{\partial p_E \partial p_I} + \frac{\partial q_E(p_I,p_E)}{\partial p_I} \geq \frac{\partial^2 A(p_I,p_E)}{\partial p_E \partial p_I} \quad (2.8)$$

The equilibrium is unique, if c^T close to zero.

Proof. See appendix.

Proposition 2.1 says that whenever the access payment function's curvature is moderate enough (neither too convex nor too concave), the game does have an equilibrium. Notice that given the assumptions on demand, we expect the l.h.s. of (2.7) to be negative, whereas the l.h.s. of (2.8) is positive. Hence (2.7) is a lower bound and (2.8) an upper bound to $\frac{\partial^2 A(p_I,p_E)}{\partial p_I \partial p_E}$.

Note again that the access payment function $A(p_I, p_E)$ is a function of both retail prices. Conditions (2.7) and (2.8) ensure that, the reactivity of the access payment function with respect to either firm's end service price is not to dependent of the other firm's retail price. In other words changes of either firm's retail price should not influence the choice of the other firm's end service price as the access payment function is concerned.

The proposition is best described by looking at a few benchmarks. Assume that

$$A(p_I,p_E) = a\, q_E(p_I,p_E),$$

i.e. E compensates I for every unit of network service used. Using (2.7) and (2.8) and rearranging yields:

$$c^T \frac{\partial^2 q_E(p_I,p_E)}{\partial p_I \partial p_E} - (p_I - c^T - c^S)\frac{\partial^2 q_I(p_I,p_E)}{\partial p_I \partial p_E} - \frac{\partial q_I(p_I,p_E)}{\partial p_E} \leq a\frac{\partial^2 q_E(p_I,p_E)}{\partial p_I \partial p_E}$$

$$(p_E - c^S)\frac{\partial^2 q_E(p_I,p_E)}{\partial p_E \partial p_I} + \frac{\partial q_E(p_I,p_E)}{\partial p_I} \geq a\frac{\partial^2 q_E(p_I,p_E)}{\partial p_E \partial p_I}$$

Given the assumptions on the demand curves, these inequalities are satisfied.

The second benchmark is the so-called Bill & Keep[9] where both firms agree to not compensate each other for the use of the competitor's network. Note that pure Bill &

[9]See Degraba (2000) for more details

Keep is unrealistic in our model of one-way interconnection, since there is only one party who has to incur cost of transmitting data, namely the incumbent. However the analysis is instructive since it is polar case that is wroth looking at.

The access payment function is now given by

$$A(p_I, p_E) = 0,$$

hence the conditions for our game to be supermodular reduce to

$$\begin{aligned}(p_I - c^T - c^S)\frac{\partial^2 q_I(p_I,p_E)}{\partial p_I \partial p_E} + \frac{\partial q_I(p_I,p_E)}{\partial p_E} - c^T\frac{\partial^2 q_E(p_I,p_E)}{\partial p_I \partial p_E} &\geq 0 \\ (p_E - c^S)\frac{\partial^2 q_E(p_I,p_E)}{\partial p_E \partial p_I} + \frac{\partial q_E(p_I,p_E)}{\partial p_I} &\geq 0.\end{aligned}$$

Whenever

$$(p_I - c^T - c^S)\frac{\partial^2 q_I(p_I,p_E)}{\partial p_I \partial p_E} + \frac{\partial q_I(p_I,p_E)}{\partial p_E} \geq c^T\frac{\partial^2 q_E(p_I,p_E)}{\partial p_I \partial p_E}$$

and the restrictions on the signs of the demand cross-derivatives are met, these conditions are satisfied.

The last case we want to look at is a payment function of the form

$$A(p_I, p_E) = T + a_1 p_I + a_2 p_E$$

where T, a_1 and a_2 are arbitrary constants taken from the set of real numbers. The analysis is analog to the one for Bill & Keep. Cross-derivatives are zero in this case and the conditions are satisfied if the restrictions above are met.

These are three cases of possible access payment functions that we encounter throughout the paper, the per-unit access charge, an access charge independent of either prices or quantities and a variable function of retail prices.

2.2.4 Social Optimum

For future reference, we compute Ramsey prices, i.e. prices that maximize social welfare subject to an industry break-even condition. In order to compute conditions for the optimal Ramsey prices we have to introduce a social welfare function $W(p_I, p_E)$. Hence the social planner has to solve the Ramsey programm given by

$$\max_{p_I,p_E} W(p_I,p_E) \quad \text{s.t.} \quad (p_I - c^T - c_I^S)q_I(p_I,p_E) + (p_E - c^T - c_E^S)q_E(p_I,p_E) = F_I + F_E \quad (2.9)$$

The programm (3.20) yields a symmetric equilibrium with $p_I = p_E = p^R$. Furthermore we assume that $p^M > p^R$, so that the market is viable.

2.3 Analysis of common Access pricing rules

Whenever services are substitutable[10] a per-unit access charge is a special case of an arbitrary access payment function dependent on service prices. Hence picking only one functional form means to circumscribe the set of possible outcomes. This section presents common per-unit access rules and shows, they can be implemented by using an arbitrary function dependent on retail prices. Because such an arbitrary function disposes of more instruments than a per-unit access charge, the set of possible outcomes is significantly larger as is the case with a per-unit access charge.

2.3.1 Cost-based Access Pricing

The notion of incremental cost is of central importance to the regulation of interconnection charges. Baumol and Sidak (1994) elaborate on different cost accounting methods and every regulatory law puts special focus on the modeling of cost of providing an incremental unit of access.

For our purpose and the sake of the argument it is sufficient to know that there exists a notion of marginal or incremental cost that is applicable to the industry. We simply continue to call that marginal cost c^T.

In this subsection we assume that a regulator's policy goal is to set access charges exactly equal to c^T. The idea is, that an outside firm is provided with the right incentives to enter and there is no discrimination between incumbent and entrant since either firm has to incur the same cost in order produce the final good.

With both I and E having the same perceived (and actual) marginal cost of production c^T of network service and product differentiation in the downstream market, their profit functions reduce to

$$\begin{aligned}\Pi^I &= (p_I - c^T - c^S)q_I(p_I, p_E) - F_I \\ \Pi^E &= (p_E - c^T - c^S)q_E(p_I, p_E) - F_E\end{aligned}$$

[10] Or prices are complements.

a duopoly with differentiated products. Each firm maximizes its profit with respect to their retail price. An equilibrium is characterized by the first-order conditions

$$(p_I^D - c^T - c^S)\frac{\partial q_I(p_I^D, p_E^D)}{\partial p_I^D} + q_I(p_I^D, p_E^D) = 0 \tag{2.10}$$

$$(p_E^D - c^T - c^S)\frac{\partial q_E(p_I^D, p_E^D)}{\partial p_E^D} + q_E(p_I^D, p_E^D) = 0 \tag{2.11}$$

with p_I^D and p_E^D denoting the duopoly prices when access is priced at cost.

Now suppose that the regulator wants to implement cost based pricing by using an arbitrary access payment function $A(p_I, p_E)$. To implement prices p_I^D and p_E^D, he has to pick $A(p_I, p_E)$'s slope conditions (i.e. the derivatives) such that (2.5) and (2.10) are equated as well as (2.6) and (2.11). Solving these equalities yields conditions

$$\frac{\partial A(p_I, p_E)}{\partial p_I} = c^T \frac{\partial q_E(p_I^D, p_E^D)}{\partial p_I} > 0 \tag{2.12}$$

$$\frac{\partial A(p_I, p_E)}{\partial p_E} = c^T \frac{\partial q_E(p_I^D, p_E^D)}{\partial p_E} < 0 \tag{2.13}$$

Note that the function $A(p_I, p_E)$ implements a point in $\{p_I, p_E\}$-space, namely (p_I^D, p_E^D). Hence any arbitrary function $A(p_I, p_E)$ satisfying (2.12) and (2.13) suffices.

A simple 3-part tariff $A(p_I, p_E) = T + a_1 p_I + a_2 p_E$ is an easy way to implement the cost based access pricing solution. The parameter a_1 is given by the r.h.s. of (2.12) and a_2 is given by the r.h.s. of (2.13). If the regulator is also concerned with equity among firms, a transfer T satisfying

$$\Pi_I(p_I^D, p_E^D) + T = \Pi_E(p_I^D, p_E^D) - T$$

yields equal profits for both firms.

In this case it is much easier to rely on a per-unit price structure because this is where cost based pricing actually comes from. It is also informationally efficient, since its implementation only requires cost information. It is also easy to see that

$$A(p_I, p_E) = a\ q_E(p_I, p_E)$$

satisfies conditions (2.12) and (2.13) where $a = c^T$ in optimum.

2.3.2 Bill & Keep

Bill & Keep, or peering (Laffont, Marcus, Rey, and Tirole (2001)) as it is also called, is an interconnection practice that is common in the Internet Industry. Large providers of Internet infrastructure (so-called backbones), exchange traffic on "a give and take" basis without any money changing hands.

By virtue of the mechanism, we expect peering to occur in two-way interconnection frameworks rather than in a one-way setting. Intuitively, two networks use this scheme once off-net traffic is sufficiently equal, i.e. a network receives as much traffic from its peer as it sends to him[11].

In a one-way access framework, the equivalent to bill & keep is charging fixed prices independent of the amount of traffic. Both payment schemes have the property that the marginal price of another unit of network services is zero. However for the ease of exposition, we relate to Bill & Keep as $A(p_I, p_E) = 0$, hence I and E's profit functions reduce to

$$
\begin{aligned}
\Pi^I &= (p_I - c^T - c^S)q_I(p_I, p_E) - c^T q_E(p_I, p_E) - F_I \\
\Pi^E &= (p_E - c^S)q_E(p_I, p_E) - F_E
\end{aligned}
$$

It is straightforward to see that the first-order conditions are given by

$$
(p_I^{bk} - c^T - c^S)\frac{\partial q_I(p_I^{bk}, p_E^{bk})}{\partial p_I} + q_I(p_I^{bk}, p_E^{bk}) - c^T \frac{\partial q_E(p_I^{bk}, p_E^{bk})}{\partial p_I} = 0 \qquad (2.14)
$$

$$
(p_E^{bk} - c^S)\frac{\partial q_E(p_I^{bk}, p_E^{bk})}{\partial p_E} + q_E(p_I^{bk}, p_E^{bk}) = 0 \qquad (2.15)
$$

Since Bill & Keep is defined as $A(p_I, p_E) = 0$ it suggests itself, that the proper mechanism to implement the outcome is given by

$$
\begin{aligned}
\frac{\partial A(p_I, p_E)}{\partial p_I} &= 0 \\
\frac{\partial A(p_I, p_E)}{\partial p_E} &= 0
\end{aligned}
$$

[11]Internet interconnection pricing is discussed in Laffont, Marcus, Rey, and Tirole (2003) and Cremer, Rey, and Tirole (2000). Degraba (2000) elaborates on Bill & Keep as an efficient interconnection agreement.

These conditions also hold for any traffic independent fixed fee and can be trivially implemented by using a per-unit access charge $a = 0$.

2.3.3 Multiproduct Monopolist

Up to now we looked at access charge rules that are either derived form a per-unit interconnection price (marginal cost access pricing) or trivially implemented by setting the access price equal to zero. Other outcomes may not be implemented so easily by using only one instrument. We show that interpreting access payment as a function of all retail prices adds an additional instrument and therefore increases the set of possible outcomes.

Suppose that I is able to freely choose any access payment function and he possesses all relevant market information including demand and cost parameters. Without any restriction, I wants to achieve the highest possible profit in the market, the multiproduct monopolist's profit analyzed in section 2.2.1. Hence his goal is to implement p_1^M and p_2^M.

He picks $A(p_I, p_E)$ such that he replicates (2.1) and (2.2). Equating (2.5) and (2.1) as well as (2.6) and (2.2) delivers conditions on $\partial A(p_I, p_E)/\partial p_I$ and $\partial A(p_I, p_E)/\partial p_E$.

Lemma 2.1. *Suppose that an incumbent is free in specifying an access payment function in a one-way interconnection model and has all relevant market information. Then his optimal choice satisfies*

$$\frac{\partial A(p_I, p_E)}{\partial p_I} = (p_E^M - c^S)\frac{\partial q_E(p_I^M, p_E^M)}{\partial p_I} > 0 \tag{2.16}$$

$$\frac{\partial A(p_I, p_E)}{\partial p_E} = c^T\frac{\partial q_E(p_I^M, p_E^M)}{\partial p_E} - (p_I^M - c^T - c^S)\frac{\partial q_I(p_I^M, p_E^M)}{\partial p_E} < 0 \tag{2.17}$$

The easiest way to implement the integrated monopolist's solution, requires I to use a simple 3-part tariff of the form $A(p_I, p_E) = T + a_1 p_I + a_2 p_E$, where a_1 is given by the r.h.s. of (2.16) and a_2 is given by the r.h.s. of (2.17). The lump sum payment T has to satisfy

$$\Pi^E - T \geq 0.$$

This tariff is only conditioned on p_I and p_E and it yields maximal industry profits. Furthermore a_1 and a_2 are constants with p_I^M and p_E^M satisfying (2.1) and (2.2).

However the result could not be replicated by using a per-unit access charge alone. The intuition is straight forward. By using a per-unit access charge, I disposes of only one instrument to solve two equations, namely (2.16) and (2.17). For the case of per-unit access charges, the l.h.s. of both equations is replaced by $a\big(\partial q_I(p_I, p_E)/\partial p_I\big)$ and $a\big(\partial q_E(p_I, p_E)/\partial p_E\big)$ respectively. Furthermore, a has to solve both equations simultaneously. This is not possible with only one instrument. In order to achieve joint monopoly outcome, at least two slope conditions and an intercept[12] are needed.

Corollary 2.1. *Suppose an incumbent is allowed to freely specify an access function $A(p_I, p_E)$. Then employing a linear per-unit access access price a is not sufficient to implement the integrated monopolist's solution.*

This result is has been highlighted in the literature in a different context. Armstrong (2002b) points out that in order to implement certain outcomes, more instruments such as an output tax are needed.

2.3.4 Social Planner

At the other end of the spectrum, the social planner's solution maximizes overall welfare. In this section we show that it is not possible to implement a socially optimal solution in an industry (with or without fixed cost) by using just a per-unit access charge.

Suppose that the social planner is solely concerned with regulating retail prices p_E and p_I such that they equal marginal cost providing service, i.e. assume that fixed cost are zero. The idea is that by regulating the access charge mechanism, the social planner is able to control retail prices. Again we are assuming full information on each side of the market[13]. In the present setting regulating prices at the level of marginal cost means $p_i = c^T + c^S$ for $i = I, E$. If we want to make I and E choose these retail prices, their first-order conditions have to satisfy

[12]The intercept is needed to allocate profits among firms. However, in order to simply generate monopoly profits, two slope conditions are sufficient.

[13]Observe that if the the social planner had all information on demand and cost, there would be not need to regulate the interconnection market. He could dictate prices in the retail market and make interconnection mandatory. But for the sake of the argument we assume that the regulatory authority is only able to intervene in the access pricing process.

$$
\begin{aligned}
(p_I^S - c^T - c^S)\frac{\partial q_I(p_I^S, p_E^S)}{\partial p_I} &= 0 && (2.18)\\
(p_E^S - c^T - c^S)\frac{\partial q_E(p_I^S, p_E^S)}{\partial p_E} &= 0 && (2.19)
\end{aligned}
$$

Note that $p_I^S = c^T + c^S$ and $p_E^S = c^T + c^S$, i.e. they solve the first-order conditions (2.18) and (2.19). We can now state the following proposition:

Lemma 2.2. *Suppose a regulator wants to implement retail prices at marginal cost by using an access payment function of the $A(p_I, p_E)$. Then this function has to satisfy the following properties:*

$$
\begin{aligned}
\frac{\partial A(p_I, p_E)}{\partial p_I} &= -q_I(c^T + c^S, c^T + c^S) + c^T\frac{\partial q_E(c^T + c^S, c^T + c^S)}{\partial p_I} && (2.20)\\
\frac{\partial A(p_I, p_E)}{\partial p_E} &= c^T\frac{\partial q_E(c^T + c^S, c^T + c^S)}{\partial p_E} + q_E(c^T + c^S, c^T + c^S) && (2.21)
\end{aligned}
$$

Again the simple 3-part tariff requires a_1 and a_2 to be equal to the r.h.s. of (2.20) and (2.21). Note that the transfer payment is zero, since we abstract from fixed costs in this scenario.

Now suppose that $F_I > 0$ and $F_E > 0$ and the regulator's goal is to solve the Ramsey program (3.20) and implement $p_I = p_E = p^R$.

Lemma 2.3. *Suppose a regulator wants to implement optimal Ramsey prices p^R and p^R by using an access payment function function of the form $A(p_I, p_E)$. Then this function has to satisfy the following properties:*

$$
\begin{aligned}
\frac{\partial A(p_I, p_E)}{\partial p_I} &= -\left[(p_I^R - c_T - c_S)\frac{\partial q_I(p_I^R, p_E^R)}{\partial p_I} + q_I(p_I^R, p_E^R) - c_T\frac{\partial q_E(p_I^R, p_E^R)}{\partial p_I}\right]\\
\frac{\partial A(p_I, p_E)}{\partial p_I} &= (p_E^R - c_S)\frac{\partial q_E(p_I^R, p_E^R)}{\partial p_E} + q_E(p_I^R,_E p^R)
\end{aligned}
$$

A linear per-unit access charge cannot implement the social planner's preferred outcome. The intuition is analogous to the multiproduct monopolist's. One variable is not sufficient to solve two equations. That is to say, a per-unit access price alone does not allow for a socially optimal solution. It has to be accompanied by additional instruments, such as taxes or subsidies.

Corollary 2.2. *Suppose a benevolent social planner wants to implement marginal cost retail pricing or Ramsey prices. Then employing a linear per-unit access price a is not sufficient.*

For cost-based access pricing and a multiproduct monopolist's solution we had $\frac{\partial A(p_I,p_E)}{\partial p_I} > 0$ and $\frac{\partial A(p_I,p_E)}{\partial p_2} < 0$. To implement the socially optimal solution, the sign of both derivatives is ambiguous. However if we assume c^T to be close to zero[14] we can conclude that there occurs a sign change for both derivatives of the social planner's access payment solution as compared to the case of the integrated monopolist.

2.3.5 Evaluation of Access Payments

By appropriately specifying the access payment function, every pricing equilibrium is implementable as an outcome of a Bertrand game between the two players. The mechanism replicates slope and level conditions of any equilibrium by matching the first-order conditions. We showed that this can be done by a fairly simple three-part tariff of the form

$$A(p_I, p_E) = T + a_1\, p_I + a_2\, p_E. \tag{2.22}$$

Because it uses three instruments[15] it is superior to using a simple per-unit access charge[16].

In this section, we restrict myself to access payment functions of the form (2.22), since they exhibit all desirable properties, that allow us to implement the discussed equilibria. Graphically, the properties of such a mechanism can be depicted by the firms' best response functions. By choosing a_1 and a_2 of (2.22), we do implement a particular point in $\{p_I, p_E\}$-space by matching slope and level conditions. This is easy to see once we have a look at figure 2.3.

Consider the reference point (p_I^*, p_E^*). This is the equilibrium outcome with Bill and Keep, i.e. $A(p_I, p_E) = 0$. Choosing a_1 and a_2 in (2.22) appropriately implements any point $(p_I, p_E) \in \mathbb{R}_+^2$. This follows immediately from the firms' first-order conditions. Applying (2.22) to (2.5) and (2.6), we obtain

[14] Note that this is a sufficient condition for uniqueness and existence of an equilibrium.

[15] Note that the transfer T ensures distributional efficiency.

[16] However it has to be noted, that a per-unit access charge in combination with an output tax does the job as well.

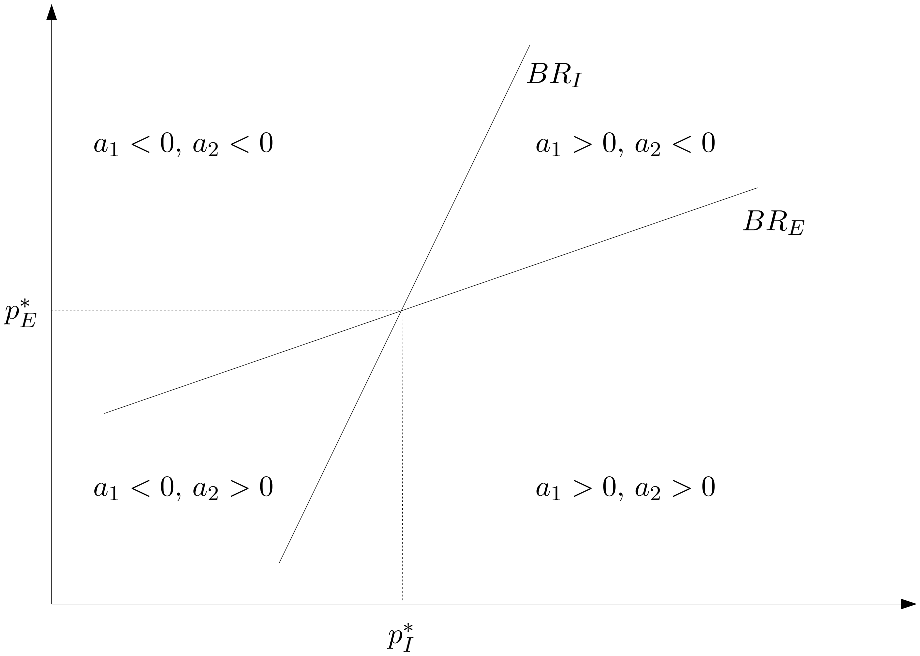

Figure 2.3: Different Regions of equilibria

$$\begin{aligned}(p_I - c^T - c_I^S)\frac{\partial q_I(p_I,p_E)}{\partial p_I} + q_I(p_I,p_E) - c^T\frac{\partial q_E(p_I,p_E)}{\partial p_I} + a_1 &= 0 \\ (p_E - c_E^S)\frac{\partial q_E(p_I,p_E)}{\partial p_E} + q_E(p_I,p_E) - a_2 &= 0\end{aligned} \qquad (2.23)$$

Because the mechanism in (2.22) yields

$$\frac{\partial^2 A(p_i,p_E)}{\partial p_i \partial p_j} = 0$$

for all $i, j = I, E$, a_1 and a_2 shift the first-order conditions. The slopes of the best response functions in (2.23) and (2.24) remain constant, hence the best response curves are only shifted.

This is depicted in figure 2.3. BR_I (BR_E) is I's (E's) best response curve, whenever $a_1 = 0$ ($a_2 = 0$). If $a_1 < 0$, I's best response is shifted to the left, for $a_1 > 0$, it is shifted to the right. Analogous for $a_2 > 0$ BR_E is shifted down and for $a_2 < 0$ up. It is certainly clear that by picking a_1 and a_2 appropriately, any point in $\{p_I, p_E\}$-space can be implemented. Furthermore, by choosing the constant T in (2.22), profits can be redistributed among the firms.

	a_1	a_2
Monopoly	$(p_E^M - c_E^S)\frac{\partial q_E(p_I^M,p_E^M)}{\partial p_I}$ $(+)$	$c^T\frac{\partial q_E(p_I^M,p_E^M)}{\partial p_E}$ $-(p_I^M - c^T - c_I^S)\frac{\partial q_I(p_I^M,p_E^M)}{\partial p_E}$ $(-)$
Cost based	$c^T\frac{\partial q_E(p_I^D,p_E^D)}{\partial p_I}$ $(+)$	$c^T\frac{\partial q_E(p_I^D,p_E^D)}{\partial p_E}$ $(-)$
Bill & Keep	0	0
Ramsey Pricing	$-\Big[(p_I^R - c^T - c_I^S)\frac{\partial q_I(p_I^R,p_E^R)}{\partial p_I}$ $+q_I(p_I^R, p_E^R) - c^T\frac{\partial q_E(p_I^R,p-_E^R)}{\partial p_I}\Big]$ $(-)$	$(p_E^R - c_E^S)\frac{\partial q_E(p_I^R,p_E^R)}{\partial p_E}$ $+q_E(p_I^R, p_E^R)$ $(+)$
Social Planner	$-q_I(p_I^{MC}, p_E^{MC})$ $+c^T\frac{\partial q_E(p_I^{MC},p_E^{MC})}{\partial p_I}$ $(-)$	$c^T\frac{\partial q_E(p_I^{MC},p_E^{MC})}{\partial p_E}$ $+q_E(p_I^{MC}, p_E^{MC})$ $(+)$

Table 2.1: Coefficients a_1 and a_2 and their respective signs

With gross substitutes, firms' best response curves are upward sloping. Using the asymmetric bill & keep equilibrium as a reference point, we can conclude, that a negative a_1 and a positive a_2 shift I and E's best response curves such that prices are smaller than the reference prices.

If prices are indicators for overall welfare, the claim is that a welfare ranking can be made by using the coefficients a_1 and a_2.

To further embark on this, let us have a look at table 2.1. It summarizes the results obtained so far, by listing the coefficients of all access pricing rules considered in the paper.

Two polar cases can be identified, namely the integrated monopolist's and the social planner's solution. The integrated monopolist is able to extract full surplus in the market and his prices are highest. The social planner fixes prices at marginal cost of production, hence there is no mark up involved.

In table 2.1 we see that a monopolist chooses $a_1 > 0$ and $a_2 < 0$. Hence he shifts out both best response functions as compared to bill & keep, resulting in higher equilibrium prices. Inspecting the social planner's choice of parameters, we realize that for c_T small[17], $a_1 < 0$ and $a_2 > 0$. With I and E's service being gross substitutes, the parameters for the cost based access price satisfy $a_1 > 0$ and $a_2 < 0$. The signs of the parameters implementing the Ramsey outcome are characterized by the following lemma.

Lemma 2.4. *In order to implement the Ramsey equilibrium by means of an access payment function of the form*

$$A(p_i, p_E) = T + a_1 p_I + a_2 p_E$$

the signs of the parameters have to fulfill $a_1 < 0$ and $a_2 > 0$ for c_T close to zero.

Proof: see appendix.

The result is summarized by proposition 2.2:

Proposition 2.2. *For c_T close to zero, the signs of the derivatives $\frac{A(p_i,p_j)}{\partial p_i}$, $i \in \{I, E\}$ pairwise indicate the desirability with respect to welfare. The combination $\frac{A(p_I,p_E)}{\partial p_I} < 0$ and $\frac{A(p_I,p_E)}{\partial p_E} > 0$ unambiguously indicates a welfare superior outcome as compared to $\frac{A(p_I,p_E)}{\partial p_I} > 0$ and $\frac{A(p_I,p_E)}{\partial p_E} < 0$.*

Proposition 2.2 gives a simple yet clear cut rule to evaluate interconnection settlements. If firms are obliged to express there interconnection agreement as a function of retail prices in the end market, the regulatory authority is able to conclude from the signs of the agreement, whether the agreement is socially desirable or subject to collusion.

We argue that this is a preliminary test. There are several reasons for that. First of all, whenever the derivatives of the access payment function exhibit the same sign, i.e. both positive or both negative, the conclusion is generally ambiguous. In other words, negative parameter for the incumbent's price is desirable for it punishes his market power. However if the coefficient on the entrant's price is negative as well, the entrant might in a favorable position, since his best response curve is shifted outwards, hence both prices

[17]In line with our assumptions on network markets.

increase[18]. In these cases the overall effect of the access payment is not revealed by the mechanism, but depends on demand as well as cost information.

However even when the coefficients are of the same sign, a regulatory authority has a first indication, where the problem of the access mechanism is hidden. In the case just described, it is a good idea, to look at the strategic positioning of the entrant. Hence by using the mechanism on retail prices as a very early indicator, underlying problems can be identified very early, hence further investigation might be easier.

Also note that the magnitude of the reported numbers does not have a meaning without information on cost and demand. Hence low numbers are not generally desirable, as well as high parameter values are not associated with socially undesirable results.

2.3.6 Bill & Keep revisited

Table 2.1 not only indicates a welfare ordering but also gives arguments in favor of Bill & Keep. The following proposition makes the point

Proposition 2.3. *Bill & Keep is welfare superior to regulating access at marginal cost for substitutable services.*

To prove this point, consider the simple model with a per-unit access charge, i.e. firms' profit functions are

$$\begin{aligned}\Pi_I &= (p_I - c_S - c_T)\ q_I(p_I, p_E) + (a - c)\ q_E(p_I, p_E) \\ \Pi_E &= (p_E - c_S - a)\ q_E(p_I, p_E)\end{aligned}$$

Given an access charge a, I and E's optimal prices are determined by the first-order conditions

$$\begin{aligned}(p_I - c_S - c_T)\ \frac{\partial q_I(p_I, p_E)}{\partial p_I} + q(p_I, p_E) + (a - c)\ \frac{\partial q_E(p_I, p_E)}{\partial p_I} &= 0 \\ (p_E - c_S - a)\ \frac{\partial q_E(p_I, p_E)}{\partial p_E} + q_E(p_I, p_E) &= 0\end{aligned}$$

[18] Notice that it is still important to look at the entrant's retail price. If the entrant's market segment is highly competitive, his retail price is close to marginal cost. In this case, the entrant is rewarded by the mechanism.

Since optimal retail prices solve these first-order conditions they are also an implicit function of the access charge. Hence applying the implicit function theorem to both equations yields p_I's and p_E's reactivity with respect to a.

$$\frac{dp_I}{da} = \frac{-\frac{\partial q_E(p_I,p_E)}{\partial p_I}}{(p_I - c_S - c_T)\,\frac{\partial^2 q_I(p_I,p_E)}{\partial p_I^2} + \frac{\partial q(p_I,p_E)}{\partial p_I} + (a-c)\,\frac{\partial^2 q_E(p_I,p_E)}{\partial p_I^2}} \tag{2.24}$$

$$\frac{dp_E}{da} = \frac{\frac{\partial q_E(p_I,p_E)}{\partial p_E}}{(p_E - c_S - a)\,\frac{\partial^2 q_E(p_I,p_E)}{\partial p_E^2} + q_E(p_I,p_E)} \tag{2.25}$$

Note that in (2.24) as well as in (2.25), the denominator has to be negative if optimality is satisfied and because services are substitutable, both derivatives are indeed positive. Hence decreasing the access charge decreases both retail prices. ■

The intuition of proposition 2.3 is straightforward. By decreasing a, E's perceived marginal cost is reduced, which puts him into a competitive advantage. Therefore he is able to reduce his equilibrium price. This is offset by an increase of the amount of service provided in equilibrium.

The more interesting case is I's decrease in retail price due to a decrease in access charge. Due to the substitutability of services, increasing p_I increases E's output. Because I provides network service to E, this has a direct effect on I's profit. Decreasing a increases the competitive pressure on I and decreases p_I. Since $(a - c^T)$, the per unit access profit (deficit) becomes smaller, increasing E's output through increasing p_I becomes more harmful in terms of I's profit. Hence I has an incentive to decrease p_I whenever a is reduced.

If prices are a perfect indicator for overall welfare[19], we can unambiguously say that bill & keep is welfare superior to regulating access prices exactly at cost in the upstream market[20].

Bill & Keep as an efficient interconnection agreement has already been discussed in the literature related to the analysis of two-way access models. Degraba (2000)[21] argues, that Bill & Keep at the central office (COBAK), reduces networks' incentives to free ride on the other network.

[19]Which is true if we assume away income effects in the social welfare function.

[20]We assume that joint profits with Bill & Keep are positive, hence cost are covered

[21]For a discussion of the COBAK see Wright (2002) and Degraba (2002)

In a one-way interconnection world, bill & keep, i.e. exchanging traffic without payment, is not rational at all. It leaves all entrants with the opportunity to free ride on the incumbent's sunk investment and hence it annihilates all of the incumbent's incentive to invest in enhancing his network.

It has been widely argued that, especially in telecommunications networks, usage based cost are negligible. The bulk of cost is sunk in the beginning. Assuming a marginal cost of transmission of zero, any per-unit access charge distorts market prices. In order to compensate the incumbent for network usage, fixed fee should be charged.

There are two arguments in favor of this change in policy. Firstly, interconnection practice evolves in line with retail prices. The past years have seen a rapid change in different network services to simple and easy flat tariffs. Only recently the German mobile provider E-Plus launched a service called BASE, where customers only paid a monthly flat fee instead of usage based prices.

Secondly, it reduces accounting cost on both sides, the regulator's as well as on the firms' side. Although cost data is still required to calculate the transfer from entrants to the incumbent. However these cost do not have to be allocated to a (possibly incorrect) estimate of demand. Using a usage independent tariff avoids some of the accounting cost, that make interconnection regulation so difficult.

2.4 Conclusion

The paper proposed a model of one-way interconnection, where both firms, the entrant and the incumbent, exhibit market power in the downstream market. Hence an access payment function based on a per-unit access charge is nothing but a mechanism based on end service prices.

By simply accounting first-order conditions we were able to reproduce results of specific access pricing rules as well as general results such as the social planners optimal retail prices or the Ramsey result by implementing an access mechanism based on retail prices.

This is due to the well known fact that the more instruments the regulator disposes of, the larger the set of possibly implementable outcomes. Armstrong (2005) recognizes the fact that using an access charge and an output tax, the social planner's preferred solution can be implemented.

By using a simple three part tariff we showed that for marginal cost of access close to zero, the derivatives with respect to the competitors prices pairwise suggest a welfare ordering. Whenever the parameter on the incumbent's price is negative and the parameter on the entrant's price is positive, prices are lower as compare to the bill & keep benchmark. This is true because both best response curves are shifted inwards, hence decreasing prices whenever services are perfect substitutes. The revers is true whenever the parameter on the incumbent's price is positive and that on the entrant's price is negative.

A similar reasoning shows that bill & keep is more efficient in terms of lower prices than an industry with access prices regulated at cost.

On a normative level the paper makes the point that using per-unit access charges as the single most important policy instrument seems to be unnecessarily restrictive. At first sight, using a per-unit access charge immediately links cost to prices, which is an a priori desirable property.

In the present model with market power on the entrant's as well as on the incumbent's side, pricing access at cost does not take care of all distortions in the market. The results of the paper show, that simple rules such as bill & keep can lead to desirable results and suggested a very simple test for regulators to judge a proposed access pricing mechanism.

Instead of using ever more technically involved methods of measuring costs in an industry, which are terribly hard to define at first and even harder to gather information of, regulatory policies should concentrate on applicability. Hence using mechanisms that require as little information as possible seems more attractive than using exact and sophisticated methods on "dirty" numbers.

The fact that litigations and debates on interconnection practices increased over the last decades, also indicates a growing need of simple rules that can be applied easily. The World Bank and the ITU termed the quest for "the" interconnection policy as the single most important problem a regulator is faced with. This is especially true with the liberalization of other network industries such as the railway network, the postal monopoly and the energy sector. Interconnection is very likely to play a big role in the regulator's policy making. Hence his toolkit should be widened rather than restricted by a single instrument.

2.A Appendix

Proof of Proposition 2.1. The idea of the proof is straightforward. In what follows we show that the game is supermodular given the restrictions on $A(p_I, p_E)$ in prop0sition 2.1. Using a theorem by Topkis, where he shows that every supermodular game has a nonempty equilibrium set, the proposition is proved.

The definition of a supermodularity is given by:

The game $(A_i, \Pi_I; i \in N)$ *is (strictly) supermodular if for each i,* A_i *is a compact lattice,* Π_i *is upper- semicontinuous and supermodular in* a_i *for fixed* a_{-i} *and displays (strictly) increasing differences in* (a_i, a_{-i})*.*

The game $(A_i, \Pi_I; i \in N)$ *is smooth supermodular if each* A_i *is a compact cube in Euclidian space,* Π_i *is twice continuously differentiable, and* $\frac{\partial^2 \Pi_I}{\partial a_{ih} \partial a_{ik}} \geq 0$ *for all* $k \neq h$ *and* $\frac{\partial^2 \Pi_I}{\partial a_{ih} \partial a_{jk}} \geq 0$ *for all* $j \neq i$ *and for all h and k. Strict inequalities for the second set of derivatives will yield strictly increasing differences in* (a_i, a_{-i}) *and a smooth strictly supermodular game.*[22]

In essence these definitions say that whenever two goods in a market are complementary in prices, that is they are substitutable and the effects increase the higher prices are, .

Most fortunate there exists a theorem by Topkis, that ensures the existence of nonempty equilibrium set for supermodular games.

Lemma A.2.1. *In a supermodular game the equilibrium set E is non-empty and has a largest,* $\bar{a} = sup\{a \in A : \bar{\Psi}(a) \geq a\}$*, and a smallest* $\underline{a} = inf\{a \in A : \underline{\Psi}(a) \leq a\}$*, element.*[23]

It remains to be shown that the game at hand is indeed supermodular:

i) The first condition is on the strategy space A_i. Notice that each players strategy is to pick a price $p_i \in [0; \infty]$, which indeed satisfies the restrictions of definition 2, where all A_i have to be compact cubes in Euclidean space.

[22] The definitions are taken from Vives (1999), page 36

[23] Again the definition is taken from Vives (1999), page 36. The proof can be found there as well

ii) The profit functions in (2.3) and (2.4) are indeed twice differentiable, which satisfies the second constraint of definition 2.

iii) The third restriction is on the second derivatives of our profit functions. Formally it can be written as

$$\frac{\partial^2 \Pi_I}{\partial p_I \partial p_E} \geq 0 \qquad \frac{\partial^2 \Pi_E}{\partial p_I \partial p_E} \geq 0 \tag{A.2.1}$$

If we apply the condition to (2.3) and (2.4), we can rewrite (A.2.1) as

$$\begin{aligned}(p_I - c_T - c_S)\frac{\partial^2 q_I(p_I, p_E)}{\partial p_I \partial p_E} + \frac{\partial q_I(p_I, p_E)}{\partial p_E} - c_T \frac{\partial^2 q_E(p_I, p_E)}{\partial p_I \partial p_E} + \frac{\partial^2 A(p_I, p_E)}{\partial p_I \partial p_E} &\geq 0 \\ (p_E - c_S)\frac{\partial^2 q_E(p_I, p_E)}{\partial p_E \partial p_I} + \frac{\partial q_E(p_I, p_E)}{\partial p_I} - \frac{\partial^2 A(p_I, p_E)}{\partial p_E \partial p_I} &\geq 0\end{aligned}$$

Rearranging yields the conditions (2.7) and (2.8).

□

Proof of lemma 2.4. The claim is

$$-\left[(p_I^R - c_T - c_S)\frac{\partial q_I(p_I^R, p_E^R)}{\partial p_I} + q_I(p_I^R, p_E^R) - c_T \frac{\partial q_E(p_I^R, p_E^R)}{\partial p_I}\right] < 0$$

$$(p_E^R - c_S)\frac{\partial q_E(p_I^R, p_E^R)}{\partial p_E} + q_E(p_I^R,_E p^R) > 0$$

Fix $c_T = 0$. Implicitly this means

$$p_I^*(p^R) > p^R \quad \text{and} \quad p_E^*(p^R) > p^R$$

where $p_i^*(p_R)$ denotes i's best response function defined by (2.10) and (2.11). Due to symmetry, $p_I^R = p_E^R = p^R$ and $p_I^D = p_E^D = p^D$, also note, that the duopoly is unique.

Suppose that $p_I^*(p^R) = p^R$. This means that $p_I^*(p_E)$ crosses the 45 degree line at $p_E = p^R$. Since the duopoly equilibrium is symmetric, both best response functions have to intersect at $p_I = p_E = p^D$. I and E's services are gross services, so $p_I^*(p_E)$ is increasing for $p_E \geq 0$. Because of Katakuni's fixed point theorem, $p_I^*(p_E)$ can intersect the 45 degree line only once, thus $p^R = p^D$. This is a contradiction, because by assumption the market is viable, hence $p^D > p^R$.

Suppose that $p_I^*(p^R) < p^R$. Note that stability requires $\frac{\partial p_I^*}{\partial p_E} < 1$, hence $p_I^*(\bar{p_E}) < \bar{p_E}$ for all $\bar{p_E} > p^D$. This implies $p^R > p^D$ if $p_I^*(p^R) < p^R$. Thus, at p^D, firms make losses, also a contradiction.

This proves the claim for $c_T = 0$. In order to complete the prove of lemma 2.4. Now increase c_T by ϵ. By continuity, we can find an ϵ small enough, for which the above claim holds. $\square$

Chapter 3

An Interconnection Agreement based on Retail Prices

3.1 Introduction

The growth of the Internet has made bandwidth and network capacity one of today's most sought after inputs. Without physical access to broadband networks, companies such as Youtube.com, Ebay.com and Google would not be possible. Similarly, firms like Skype buy network capacity to provide "old fashioned" voice services by using new technologies like Voice over IP (VoIP).

Despite the tremendous effort that has been undertaken in the nineties to liberalize telecommunications markets, facilities-based competition is still far from being implemented in most countries of the world. The former monopolists still enjoy market power by owning a significant bottleneck. In order to ensure access to the incumbent's network regulatory authorities still watch over the market for interconnection. The same is true for the utilities, railway and postal service industry.

More generally, the optimal regulation of a vertically integrated incumbent selling network access as an essential input to his competitors is still one of the most important issues in regulatory policy[1]. The answers proposed by economic literature[2] share the common assumption of a per-unit access charge. The Efficient Component Pricing Rule (ECPR) derives the interconnection charge from the incumbent's opportunity cost of providing access. The optimal Ramsey access charges are dependent on the demand structure and the physical cost of providing access. All approaches require detailed cost information and a sound knowledge of the underlying demand parameters, which is hardly accessible.

In this paper we take a new road to interconnection pricing, which we call "Parapricing". We employ a model of one-way interconnection, where an incumbent and an entrant produce two differentiated services. These services use a physical network owned by the incumbent as an input. Because products are differentiated, an access payment based on a per-unit charge is a function of both firms' retail prices. Conditioning the access payment on two retail prices allows for an additional instrument as compared to a single per-unit access price.

With Parapricing, the regulator uses this additional instrument and introduces a game between an incumbent and an entrant. In a first stage, each firm simultaneously chooses one of two distinct parameters of a linear access contract based on the both firms' retail prices. In particular the incumbent determines the weight on the entrant's retail price and the entrant the weight of the incumbent's price. In the second stage, firms choose their retail prices given the parameters of the access payment function.

This approach has two remarkable informational benefits. Firstly, it diminishes the role of the regulator and subsequently reduces the amount of information needed. With Parapricing, the regulator neither picks retail nor access prices, as is the case with most proposals in interconnection pricing. He rather acts as a supervisor that oversees the rules of the game.

Secondly, it shifts responsibility to the entrant. He is actively engaged in the interconnection pricing process. Hence decision rights are shifted to the player who actually disposes of information needed for setting prices, i.e. cost and demand information.

[1]Although the adjacency to network industries is apparent, the model can be employed to traditional industries as well. We may think of a producer of tower cranes who rents his cranes to construction firms and at the same time sells his products to competitors who also rent cranes.

[2]See Armstrong (2002b) and Laffont and Tirole (2000) for extensive surveys

After introducing Parapricing and stating its rules, we show in a simple model with linear demands, that there exists a unique equilibrium in prices and regulatory parameters. In equilibrium prices are lower and welfare is higher as compared to pure cost-based access pricing. This is especially appealing to markets with differentiated products, since in this setting, retail prices are not only distorted via the access market, but also due to imperfect competition downstream. Furthermore, we derive conditions for which Parapricing yields the same results as the Ramsey program.

The analysis of the paper suggests that Parapricing's decision structure creates a trade off between increasing/decreasing profits directly via the access parameter and indirectly via resulting changes in retail prices. As compared to regulating access at marginal cost, the incumbent has an incentive to increase its access revenues, thereby by decreasing both retail prices. The entrant on the other hand wants to decrease his access payment and thereby decreases both retail prices. This yields overall lower prices as in the case of symmetric duopoly, i.e. cost based access pricing.

The paper is organized as follows. Section 3.2 reviews the results and assumptions of the literature on per-unit access charges. Section 4.2 introduces the model and the regulatory framework. The equilibrium is derived in section 3.4. Section 3.5 compares the results to important benchmarks. The general framework and strengths and weaknesses in terms of applicability are discussed in section 3.6. Section 4.5 concludes.

3.2 What, why and how to regulate?

Officially telecommunications markets in most industrialized countries have been "fully liberalized"[3] in the mid nineties. Effectively former state owned monopolists are still subject to regulatory laws. In most countries the incumbent is the sole proprietary of a physical network, which is an essential input for the entrants' services. Hence the incumbent has a natural monopoly in the upstream market. This raises concern that the incumbent exercises his market power in order to exclude entrants or gain supranormal

[3] The European Commission states that "Since 1 January 1998, the telecommunications markets are fully liberalised in most of the European Union." (European Commission (1999)). Cite FCC Telecommunciations Act 1996.

profits. Therefore markets for interconnection are regulated.

Laws like the Directive 97/33 of the European Commission provide National Regulatory Authorities (NRA) and market agents with a framework within which interconnection settlements are to be reached. Amongst other things, it states procedures how to define a relevant market [4], dispute settlement processes and cost accounting procedures.

The access payment mechanism as such is never defined explicitly. For instance the FCC states in its FCC Telecommunications Act 1996 (Part II, Section 251):

> "The duty to provide, for the facilities and equipment of any requesting telecommunications carrier, interconnection with the local exchange carrier's network on rates, terms, and conditions that are just, reasonable, and nondiscriminatory, in accordance with the terms and conditions of the agreement and the requirements of this section and section 252."

Nothing is said about either the structure or the quantitative measure of interconnection agreements. They only have to be "just, reasonable, and nondiscriminatory".

The EU encouraged its member states to transfer the responsibility of finding an interconnection mechanism to the firms. If they fail, every NRA has to implement dispute settlement procedures in order to reach mutual consent among parties. This requires active guidance by regulators to find an optimal access payment mechanism which is borne by firms and their customers.

An optimal access payment mechanism has to achieve at least two goals at the same time: on the one hand it has to increase competition by inducing (efficient) entry. On the other hand the network proprietary has to be provided with enough incentives to invest in infrastructure. The ultimate goal is to guarantee a sufficient (maximal) level of welfare to consumers and producers alike.

As long as market characteristics such as demand and cost structure are known to the regulator, this is an optimization problem, which is solvable — at least numerically. Unfortunately, the regulator does not dispose of the required information. He has little knowledge, if any, about cost and demand structures in the relevant industries. Mostly firms even have incentives to hide their cost structure.

[4] A description of the market definition process and an extensive discussion can be found in Gual (2003).

Hence the optimal regulatory strategy has to be found in the absence of information of the regulatory authority. In that sense economic theory should always be concerned with a high degree of practicability, since obtaining information about firms, consumers and markets in general is timely and costly to gather.

Let us have a look at the solutions suggested by the literature on interconnection to solve the regulator's problem. Among the first to address the question were Willig (1979) and Baumol (1983). They introduced the notion of an access charge, i.e. a per-unit price paid by the entrant in order to use the incumbent's facilities.

Most of the earlier papers assumed perfect competition in the downstream market. Hence, it suffices to identify the marginal cost of providing access, because that determines the optimal access price. Simply setting the price equal to marginal cost ensures the socially optimal outcome in the market.

However due to the inherent fixed costs, that are associated with setting up and running a fully fledged network, the literature adopted broader notions of costs. Baumol and Sidak (1994) and Laffont and Tirole (2000) distinguish between marginal, incremental, average incremental cost, cost in the long-run or the short-run.

One thing, all of the cost definitions have in common is that they are hard to measure for any regulatory authority. Accurate information about average incremental cost or marginal cost can only be provided by network owners themselves. When it comes to cost forecasts, even firms may fail to provide correct numbers.

Incentive regulation always has to deal with inherent asymmetric information[5]. Regulatory laws on interconnection include detailed cost accounting procedures to circumvent this problem. However because the incumbent has strong incentive to exaggerate his cost, effort is undertaken to report favorable figures to the NRAs[6]. Hence cost information the regulator disposes of is at best questionable, as is its application to access price regulation.

These are direct cost measures, that determine the access price in a "quid pro quo"

[5]The classic citation for incentive regulation is certainly Laffont and Tirole (1993) who discuss the literature extensively.

[6]Incentive regulation proposes mechanisms that deal with this asymmetry in particular. These bring together incentives to truthfully report cost parameters as well as to increase efficiency of production. The reader is referred to Laffont and Tirole (1993) and the discussions on price cap and rate of return regulation.

manner. A second class of access charge rules are often referred to as usage-based (Laffont and Tirole (1996)). One of them, the ECPR, uses the opportunity cost of the incumbent due to the entry of another firm into its market. Armstrong (2002b) defines the ECPR as

$$\begin{aligned} access\ charge &= cost\ of\ providing\ access \\ &\quad + incumbent's\ lost\ profit\ in\ retail\ markets \\ &\quad caused\ by\ providing\ access. \end{aligned}$$

The ECPR permits only efficient entry, i.e. by firms operating at a lower cost than the incumbent[7]. Despite its intuitive appeal and the beauty of simplicity, several objections have been brought forward. Economides and White (1995) point out that the efficiency of the ECPR is only given at a very specific set of assumptions. One of them is the assumption that the pricing of the monopolist reflects marginal costs. They show that as soon as the monopolist exercises market power, the ECPR eventually protects supranormal profits. Ergas and Ralph (1994) highlight the fact that from a cost of regulation point of view nothing is gained as compared to cost-based pricing. They adopt the view that the ECPR is as costly as the Ramsey pricing in terms of information. The application of the ECPR requires explicit knowledge of the demand and cost structure in the market.

In two seminal papers, Laffont and Tirole (1994) and Armstrong, Doyle, and Vickers (1996) relax the assumption of perfect competition in the downstream market. They employ a model with an incumbent exhibiting market power and a competitive fringe and derive expressions for the optimal Ramsey access charge. Ramsey access charges exhibit a mark-up on marginal cost because of the inherent fixed cost in the industries. They are also usage-based, hence optimal prices are determined by the elasticities of the demand functions and the marginal cost of providing access. Therefore implementing Ramsey prices requires detailed knowledge of demand and cost parameters as is the case with ECPR.

A way to overcome these informational barriers are global price caps[8]. The idea is

[7] Numerical examples are provided by Ergas and Ralph (1994)) as well as Economides and White (1995)

[8] Global and partial price caps are treated extensively in Laffont and Tirole (1996), Laffont and Tirole (1994) and Vogelsang (2003)

straightforward: instead of fixing prices at a particular value, the portfolio of prices, including the access charge, is restricted in terms of its relative structure and its overall level. It can be shown, that choosing appropriate weights for a global price cap results in firms choosing optimal Ramsey prices. These weights turn out to be output levels produced at Ramsey prices. Note that with this approach, neither elasticities nor cost information is needed. However, weights have to be correctly anticipated. In practice[9], past output levels are used.

Although the latter approach reduces the amount of information considerably, the regulator is still intervening actively in firms' pricing decision. As is the case with all proposals discussed above, this requires information that is hardly accessible. Additionally, market parameters such as elasticities have to be computed as accurately as possible. This is both time and money consuming, sometimes even impossible. Finally defining accurate notions of cost or markets, may turn out difficult in rapidly evolving markets such as telecommunications markets. This questions the applicability of traditional per-unit access charge rules.

With Parapricing we take this fundamental scarcity of available information into account.[10] The regulator only determines the rules of the regulatory game played by the firms. It is a mechanism based on both retail prices, that of the incumbent and the entrant. Within the framework, firms are free to determine prices and the access parameters.

3.3 The Model

In this section we specify the model, that underlies the remainder of the paper. The incumbent I is the sole proprietary of a physical network[11]. To produce service in the downstream market, both I and an entrant E use network capacity as an essential input.

[9] For a discussion on practical issues and experience see Laffont and Tirole (1996) and Crew and Kleindorfer (1996)

[10] To our knowledge Jeon and Hurkens (forthcoming) is the only paper accounting for this lack of information by designing an appropriate mechanism. Laffont and Tirole (1994) use an asymmetric information approach, however the regulator still has to dispose of information on the cost distribution.

[11] The model derived with the telecommunications industry in mind, where the essential facility is a fiber/copper/mobile network. Notice however, that this could be any technology physical or not, that happens to have analogous market characteristics

A unit of network capacity is required to produce a unit of the final product. Entry is not facility based, hence the entrant will not duplicate the entire network, because the fixed cost is prohibitively high.

■ **Demand structure:** An incumbent and an entrant compete in a downstream service market offering imperfect substitutes. To make the model tractable we assume linear demand functions of the form

$$q_i(p_i, p_j) = 1 - p_i + \sigma p_j \quad \forall\, i \in \{I, E\} \;\; i \neq j \tag{3.1}$$

where $\sigma \in]0, 1[$. Note that the derivatives satisfy $\frac{\partial q_i(p_i,p_j)}{\partial p_i} < 0$, $\frac{\partial q_i(p_i,p_j)}{\partial p_j} > 0$, $\frac{\partial^2 q_i(p_i,p_j)}{\partial {p_i}^2} \geq 0$ and $\frac{\partial^2 q_i(p_i,p_j)}{\partial p_i \partial p_j} \geq 0$ for all $i \in \{I, E\}$. Hence products are substitutes and prices strategic complements[12].

We do not model demand as a subscription demand, as for example de Bijl and Peitz (2004) do. Consumers do not decide whether to be part of a network, but how much of the product to buy. We strongly believe that this is justified in "new" types of content networks markets, as well as data networks.

■ **Cost structure:** Overall cost is determined by production in upstream and downstream markets. For the transmission of one unit of data, or more generally for one unit of network usage, I incurs a constant marginal cost of c_T.

In the downstream market, I and E incur a constant marginal cost c_S for producing one unit of service. In traditional telecommunications markets it is often referred to as the cost of switching, i.e. the cost a firm has to incur at a central or regional switch for completing a connection of two agents. Naturally, c_S could differ across firms.

Furthermore we assume that every unit of final service requires exactly one unit of network good and one unit of switching. Hence, the actual cost of providing one unit service to customers amounts to $c_S + c_T$. For simplicity and ease of presentation we assume $c_S + c_T = 2c$ for the rest of the paper.

[12]Demand functions are derived from the maximization of a net-utility function given by

$$U(q_I, q_E) = -\frac{1}{2(1-\sigma^2)}(q_I^2 + q_E^2) - \frac{\sigma}{(1-\sigma^2)} q_I q_E + \frac{1}{1-\sigma}(q_I + q_E) - p_I q_I - p_E q_E$$

which is quadratic and strictly concave given the restriction on σ.

Scenario	I	E	Regulator
A	a_1, a_2	—	—
B	—	a_1, a_2	—
C	a_1	a_2	—
D	—	—	a_1, a_2
E	a_2	a_1	—

Table 3.1: Possible decision structures

In addition to the variable cost, both firms incur a fixed cost of F_I and F_E.

■ **Regulatory setting:** Parapricing takes the regulator's lack of information explicitly into account. This is not in the sense of incomplete information, but rather as a complete lack of information of either firms' cost or consumers' demand. In the absence of effective regulation, the incumbent tries to maximize profits and skim off all the profits that can be realized in the downstream market. The regulator has to specify an environment where entry is possible and firms compete effectively. Ideally, this framework is as general and easily verifiable as possible. As discussed above, if downstream services are differentiated, the access charge payment function is a special case of a general access payment function based on I and E's retail prices. The access payment mechanism used in this paper is a general function of retail prices given by

$$A(p_I, p_E) = T + a_1 p_I + a_2 p_E. \tag{3.2}$$

The access payment function consists of five different variables that have to be decided upon. We want firms to set their own prices, that is I picks p_I and E picks p_E. Leaving aside the parameter T for the moment[13], a_1 and a_2 have to be allocated to either the firms or the regulator as a decision variable. Table 3.1 summarizes five possible decision structures for the parameters of the linear access payment function.

[13]We will turn to that in section 3.6

In scenario A and B, either I or E decide upon the parameters of the regulatory game. The advantage is that the regulator does not have to intervene in the market. Hence he does not have to dispose of any market information.

However if the incumbent sets both a_1 and a_2, he is able to extract joint industry monopoly profits. If E decides on a_1 and a_2, it can be shown that there only exists a cornered equilibrium. Therefore we can rule out scenarios A and B as a desirable mechanism.

In scenario C, either firm decides upon its "own" parameter. From the point of view of the regulator, it is desirable, since no information is needed. It is the sole responsibility of both firms to determine the access payment contract. However in terms of lower prices and enhanced competition, nothing is gained. To see that let us look at the first-order conditions of the pricing game.

$$(p_I - 2c)\frac{\partial q_I(p_I, p_E)}{\partial p_I} + q_I(p_I, p_E) - c\frac{\partial q_E(p_I, p_E)}{\partial p_I} + a_1 = 0 \tag{3.3}$$

$$(p_E - c)\frac{\partial q_E(p_I, p_E)}{\partial p_E} + q_E(p_I, p_E) - a_2 = 0 \tag{3.4}$$

From (3.3) and (3.4) it is apparent that if I determines a_1 and E decides upon a_2, both firms are able to shift their best response functions outwards, resulting in higher end user prices. This is reminiscent of the "raising each others cost" practice in the two-way access pricing problem discussed in Laffont, Rey, and Tirole (1998a) and Armstrong (1998).

In scenario D of table 3.1, the regulatory authority chooses a_1 and a_2. Notice that there is no substantial informational gain to Ramsey pricing with a per unit access charge[14]. Both possibilities imply that the benevolent social planner has to have enough knowledge on demand and cost parameters. In that sense, no mechanism is superior over the other.

Finally in scenario E I determines a_2 and E chooses a_1. Notice that through (3.3) and (3.4), their choices have direct consequences on the retail pricing decision of their competitor. Since the regulator does not determine any parameter of the access payment function, this mechanism is certainly superior in terms of information.

By designing a game this way, we show that I (E) faces a trade off between increasing

[14]In chapter 2 of this monograph, it is shown that there is indeed a difference between both scenarios. With scenario D, the regulator is actually able to implement the social optimum where retail prices are set to marginal cost. This result is due to the additional instrument. However if a per-unit access charge is combined with an output tax, both approaches are equivalent.

	I	E	Regulator
Retail	$p_I \geq 0$	$p_E \geq 0$	—
Access	$a_2 \geq 0$	$a_1 \leq 0$	T

Table 3.2: Parapricing's strategic variables

own profit through increased (decreased) access payment and decreasing retail prices. We also show that, reducing retail prices results in a comparative disadvantage for I.

Furthermore, we assume that $p_I \geq 0$ and $p_E \geq 0$. It is shown in the appendix that this avoids peculiar pricing schemes that could possibly occur. Intuitively for $a_1 \leq 0$ it is possible that I prefers to set a negative price in order make his compensation payment positive. The same is true for E. With $a_2 > 0$ setting $p_E < 0$ lets E enjoy an access transfer. Table 3.2 summarizes the strategic variables in the regulatory game and their allocation.

To sum up, the regulation mechanism is modeled in two stages. First, I picks $a_2 \geq 0$ and E sets $a_1 \leq 0$ in order to determine (3.2). In a second stage, firms pick their non-negative retail prices.

Notice that the role of the regulator reduces considerably as compared to the regulation schemes discussed in section 3.2. It suffices to set up the framework, i.e. the access payment mechanism and to choose the entrants that are allowed to play the game[15].

Having defined the strategic variables of each firm, we now turn to the analysis of the firms' decision problem.

■ **Firms' problem:** In line with the preceding section, the profit functions for I and E are given by

$$\Pi^I = (p_I - c)q_I(p_I, p_E) - c(q_I(p_I, p_E + q_E(p_I, p_E)) + a_1 p_I + a_2 p_E - F_I \quad (3.5)$$

$$\Pi^E = (p_E - c)q_E(p_I, p_E) - a_1 p_I - a_2 p_E - F_E \quad (3.6)$$

This fully characterizes a two player game in two stages:

[15] The role of the regulator in choosing the entrant is discussed in section 3.6

1. Firms choose the regulatory parameters a_1 and a_2, where I decides upon a_2 and E decides upon a_1.

2. I and E determine p_I and p_E respectively.

The game is solved by backwards induction. The next section characterizes the equilibrium outcome.

3.4 Equilibrium Analysis

Solving the game for a subgame perfect equilibrium leads to the first result of the paper:

Proposition 3.1. *Consider the two-stage game defined by (3.5) and (3.6) where $q_I(p_I, p_E)$ and $q_E(p_I, p_E)$ are given by (3.1). In the first stage firm I picks $a_2 \geq 0$ and firm E picks $a_1 \leq 0$, In stage two retail prices $p_I \geq 0$ and $p_E \geq 0$ are determined by I and E respectively.*

For marginal cost sufficiently low and close enough substitutes, the unique equilibrium of this game is given by:

$$p_I^*(a_1^*, a_2^*) = \frac{1}{2-\sigma} + \frac{c(4-\sigma)}{4-\sigma^2} + \frac{2a_1^* - \sigma a_2^*}{4-\sigma^2} \tag{3.7}$$

$$p_E^*(a_1^*, a_2^*) = \frac{1}{2-\sigma} + \frac{c(\sigma(2-\sigma)+2)}{4-\sigma^2} + \frac{\sigma a_1^* - 2a_2^*}{4-\sigma^2} \tag{3.8}$$

where a_1^* and a_2^* are given by

$$a_1^* = \frac{\sigma^3 c - \sigma^2 - 2\sigma^2 c - 2\sigma - 4\sigma c + 4 + 8c}{\sigma^2 + 4\sigma - 8} \tag{3.9}$$

$$a_2^* = \frac{-8\sigma c + 5\sigma^2 c - 4 + 2\sigma + \sigma^2}{\sigma^2 + 4\sigma - 8}. \tag{3.10}$$

Proof. See appendix.

Note that the $p_I^*(a_1^*, a_2^*)$ and $p_E^*(a_1^*, a_2^*)$ are finite, nonzero and symmetric. To see this evaluate the equilibrium prices at the equilibrium parameters a_1^* and a_2^*.

To understand firms' strategic interaction when choosing the regulatory parameters, we have to disentangle the effects of changes in the parameters with respect to retail prices and overall profits. Computing the first-order conditions of each firm with respect to its regulatory parameter, we obtain

$$\begin{aligned}(p_I - c)\left[\frac{\partial q_I}{\partial p_I}\frac{\partial p_I}{\partial a_2} + \frac{\partial q_I}{\partial p_E}\frac{\partial p_E}{\partial a_2}\right] + \frac{\partial p_I}{\partial a_2} q_I(p_I, p_E) \\ = -\left[a_1 \frac{\partial p_I}{\partial a_2} + p_E + a_2 \frac{\partial p_E}{\partial a_2} - c\left(\frac{\partial q_E}{\partial p_I}\frac{\partial p_I}{\partial a_2} + \frac{\partial q_E}{\partial p_E}\frac{\partial p_E}{\partial a_2} + \frac{\partial q_I}{\partial p_I}\frac{\partial p_I}{\partial a_2} + \frac{\partial q_I}{\partial p_E}\frac{\partial p_E}{\partial a_2}\right)\right]\end{aligned} \tag{3.11}$$

and

$$(p_E - c)\left[\frac{\partial q_E}{\partial p_I}\frac{\partial p_I}{\partial a_1} + \frac{\partial q_E}{\partial p_E}\frac{\partial p_E}{\partial a_1}\right] + \frac{\partial p_E}{\partial a_1} q_E(p_I, p_E) = a_1\frac{\partial p_I}{\partial a_1} + p_I + a_2\frac{\partial p_E}{\partial a_1} \tag{3.12}$$

Equations (3.11) and (3.12) characterize the optimality conditions for a_1 and a_2. At the margin, each firm chooses its parameter such that a change in retail profit due to changes in retail prices is offset by the change in access profit/loss.

Consider the entrant's choice of a_1. Changing a_1 has two effects on E's profit: (i) directly via the parameter in the profit function and (ii) indirectly by changing equilibrium prices. Because of (3.12) the direct marginal effect on E's profit is $-p_I$[16]. Hence decreasing a_1 by an infinitesimal amount increases his profit by exactly the price of one unit of service.

The indirect effect of changing a_1, that changes firms' retail prices and demands, is identified by differentiating (3.7) and (3.8) with respect to a_1:

$$\frac{\partial p_I}{\partial a_1} = \frac{2}{4-\sigma^2} \tag{3.13}$$

$$\frac{\partial p_E}{\partial a_1} = \frac{\sigma}{4-\sigma^2} \tag{3.14}$$

$$\frac{\partial q_I}{\partial a_1} = -\frac{2-\sigma^2}{4-\sigma^2} \tag{3.15}$$

$$\frac{\partial q_E}{\partial a_1} = \frac{\sigma}{4-\sigma^2} \tag{3.16}$$

Because the access payment function is linear in both firms' retail prices, increasing a_1 shifts I's best response function in the pricing game outwards. Because of (3.3) and (3.4) I's best response $p_I^*(p_E)$ is increased for every p_E ceteris paribus, i.e. the best response function is shifted outwards. However changing a_1 does not alter E's best response function. Graphically this is depicted in figure 3.1.

Shifting I's best response curve implies that equilibrium prices have to evolve along E's best response curve. This also determines the relative change of p_I and p_E with respect to a_1. For the case of linear demands, the slope of E's best response function is given by

$$\frac{dp_E}{dp_I} = -\frac{(p_E - c)\frac{\partial^2 q_E}{\partial p_I \partial p_E} + \frac{\partial q_E}{\partial p_I}}{(p_E - c)\frac{\partial^2 q_E}{\partial p_E^2} + 2\frac{\partial q_E}{\partial p_E}} = \frac{\sigma}{2},$$

[16]Note that all prices are evaluated at their equilibrium value. Hence each price is essentially a function of a_1 and a_2.

which is less than one for $\sigma \in]0,1[$. Increasing a_1 makes I's service relatively more expensive than E's service. This is apparent from (3.13) and (3.14). The reverse is true for $a_1 < 0$. This makes I's service relatively cheaper as compared to E's service.

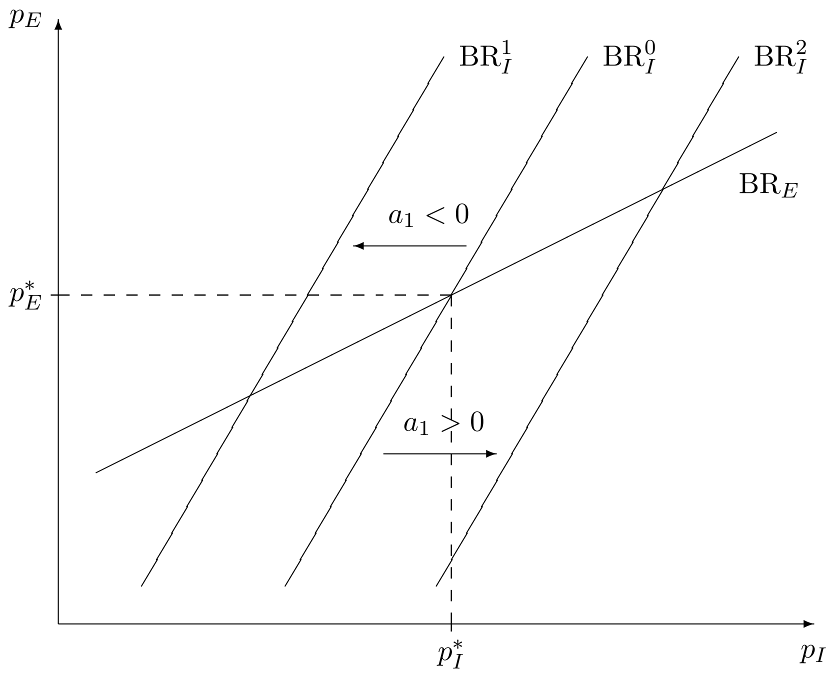

Figure 3.1: Best response curves with different a_1's

Because the products are imperfect substitutes, the effect on demand is generally ambiguous. For the case of symmetric linear demands, (3.15) and (3.16) suggest, that increasing a_1 reduces I's and increases E's realized demand. Hence increasing a_1 puts the entrant in a relative competitive advantage. It makes I's service relatively more expensive, which induces some buyers to "switch and others to stop buying all together. Overall, increasing a_1 decreases joint realized demand. In sum, decreasing (increasing) a_1 decreases (increases) p_E and q_E, hence decreases (increases) E's retail profit.

Now consider the access payment and fix $a_1 = a_2 = 0$. In this case, the marginal effect of changing a_1 on E's access payment is simply $-p_I$. By decreasing a_1 below zero, E exhibits access revenues, i.e. he puts his competitor in a better position by making his product relatively cheaper but takes his access profit away through decreasing" his own payment. E's retail profit is decreased by the decrease in a_1. In equilibrium, these effects offset each other, so that the marginal loss of retail profit is offset by the access payment received from the incumbent.

Now let us look at I's choice of a_2. We identify direct and indirect effects and look at their impact on I's retail and access profits. Suppose I increases a_2 to a_2'. In order to illustrate the impact of this increase graphically, we isolate the direct effect in figure 3.2.

A is I's initial isoprofit curve with a_2 . Keeping retail prices constant, increasing a_2 increases I's profit directly by $(a_2' - a_2)p_E^*$[17]. Because we only look at the direct effect, neither I's nor E's best response curve have shifted, i.e. prices remain constant at p_I^* and p_E^* in figure 3.2.

The direct effect increases I's profit but has no impact on prices. Hence I's new isoprofit curve A'' at a_2' has to pass through p_I^* and p_E^*. Because p_I^* is still optimal for I, the slopes of A and A'' are both zero at p_I^*. Off the equilibrium, increasing a_2 widens the isoprofit curve A''.

The incumbent's isoprofit curve A' illustrates the increase in profit due to the increase in a_2. Increasing p_E^* to p_E' leaves I with the same profit as changing a_2 to a_2'. Increasing E's price with p_I fixed gives I a relative competitive advantage and induces an increase in I's demand. I is indifferent between increasing p_E to p_E' with constant a_2 and p_I^* and a flat payment of $(a_2' - a_2)p_E^*$ with constant retail prices.

Changing a_2 changes the shape of the isoprofit curve but not the location of the pricing equilibrium. Varying p_E with a_2 constant shifts the isoprofit curve without changing its shape.

Now consider the change in equilibrium prices due to a change in a_2. This is the indirect effect in figure 3.3. Because downstream service are imperfect substitutes, the best response function of either firm is upward sloping. By increasing a_2, E's best response function is shifted downwards (BR_E to BR_E'). Looking at (3.4) it is immediate, that an increase in a_2 does not change the slope of E's best response functions since it is independent of a_2. Hence a change in a_2 changes only the location of E's best response curve in (p_I, p_E)-space. I's best response curve is independent of a_2, hence remains unchanged.

Equilibrium prices decrease from p_I^* and p_E^* to p_I^** and p_E^**. I's final profit level after

[17]In order to isolate the direct effect, $(a_2' - a_2)p_E^*$ can be thought of as a constant flat payment.

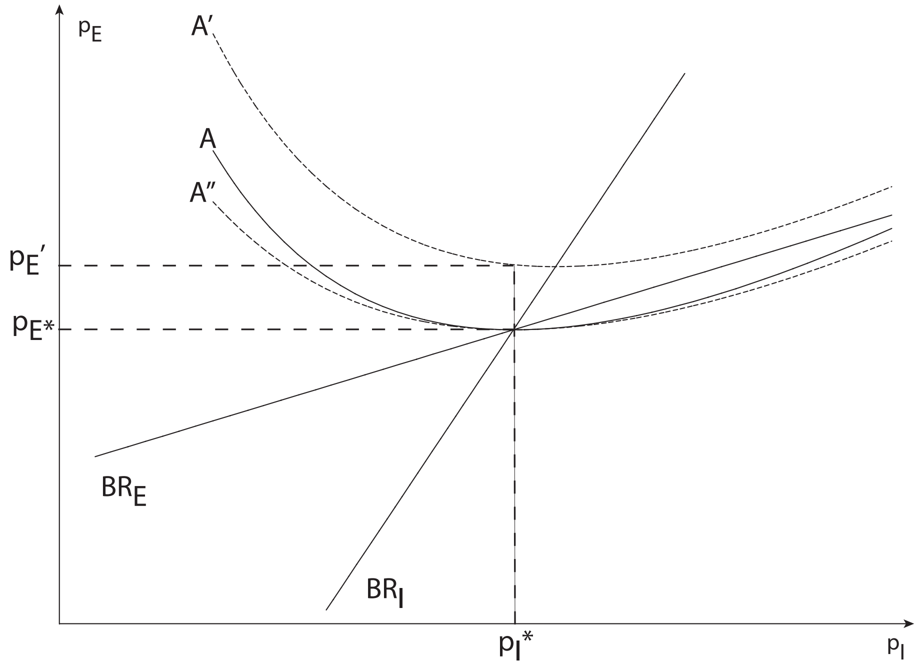

Figure 3.2: Direct Effect

the change in a_2 is indicated by AA'.

Implicit in the isoprofit curve A'' is the direct effect on I's profit, due to the change in a_2. This makes it hard to compare the new to the original equilibrium in terms of isoprofit curves.

The indirect effect is illustrated by the isoprofit curves AA and AA'. Curve AA is the analogue to A' in figure 3.2, with $\Pi_I = \Pi_I(p_I^*, p_E', \bar{a_2})$[18].

Proposition 3.1 states, that in equilibrium, prices will be symmetric. Note that p_I^* and p_E^* are the direct effects on firms' profits of a change in a_1 and a_2. In equilibrium these have to be equal to the changes in profit due to the price changes, that arise because of the variation in a_1 and a_2. In a symmetric equilibrium, these marginal effects on either firm's profit are identical.

This is a particularity of the symmetric linear demand model. It is easy to verify that the lack of second order effects of prices on demand implies equally sloped best response curves. Furthermore this implies symmetric derivatives of prices with respect to the parameters a_1 and a_2.

[18]Note again, that $\Pi_I(p_I^*, p_E', \bar{a_2}) = \Pi_I(p_I^*, p_E^*, \bar{a_2}')$

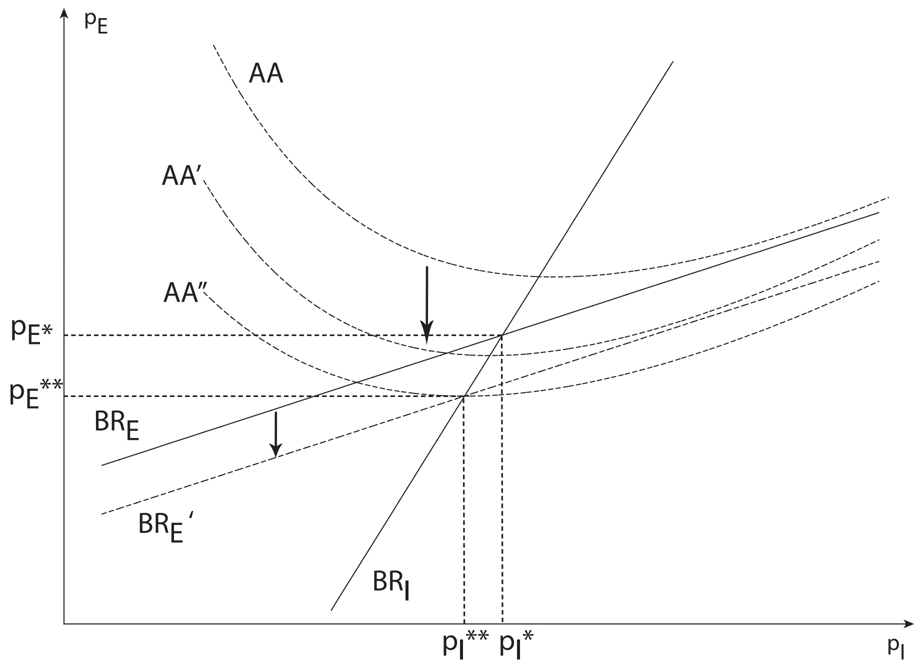

Figure 3.3: Indirect Effect

3.5 Benchmarks and Extensions

Ultimately we want to compare Parapricing's performance to its alternatives, i.e. the access charge regulated at marginal cost on the one hand and the optimal Ramsey outcome on the other hand. This subsection puts our mechanism to the test. In case of cost based regulation of per-unit access charges, the regulator knows the marginal cost of providing access. The second benchmark is the Ramsey outcome. We compare the results to the ones obtained in the previous section and derive orderings according to prices and welfare.

In order to be able to compute explicit solutions, we employ the linear demand framework introduced in 4.2. The analysis is extended in that we allow for asymmetric linear demand systems with captive buyers and different levels of substitutabilities.

3.5.1 Cost-based Regulation

With cost-based regulation, the regulator sets a per-unit access charge equal to cost. In our model, $A(p_I, p_E) = a\ q_E(p_I, p_E)$ with $a = c$. Firms maximize profits by choosing optimal prices. It is easy to see that these are given by

$$p_I^a(c) = p_E^a(c) = \frac{1+2c}{2-\sigma} \tag{3.17}$$

Using the equilibrium prices and the utility function for the linear case we can state the following proposition

Proposition 3.2. *Equilibrium prices under Parapricing are lower and welfare is higher than under a regulated per unit access price at exactly the marginal cost of providing access.*

Proof. See appendix

By using an already (perfectly) regulated per unit access price we put a harsh informational restriction on the regulator. In reality, the regulator is hardly ever able to compute the marginal cost of providing access that accurate.

On the other side Parapricing leaves all relevant decisions to the firms. The only task remaining for the regulator is to make the firms play the game and compute the parameters a_1 and a_2 independently and non-cooperatively.

The intuition for this result is straightforward. In order to see this, consider again the first-order conditions in (3.3) and (3.4):

$$\begin{aligned} FOC_I^{a_1} &= (p_I - 2c)\frac{\partial q_I(p_I, p_E)}{\partial p_I} + q_I(p_I, p_E) - c\frac{\partial q_E(p_I, p_E)}{\partial p_I} + a_1 \\ FOC_E^{a_2} &= (p_E - c)\frac{\partial q_E(p_I, p_E)}{\partial p_E} + q_E(p_I, p_E) - a_2 \end{aligned}$$

Let us compare these to the first-order conditions that result from the per-unit access fee model. Using the linear demand functions in (3.1), differentiating symmetric duopoly profits with in p_I and p_E yields

$$FOC_I^{a=c} = (p_I - 2c)\frac{\partial q_I(p_I, p_E)}{\partial p_I} + q_I(p_I, p_E) \tag{3.18}$$

$$FOC_E^{a} = (p_E - c)\frac{\partial q_E(p_I, p_E)}{\partial p_E} + q_E(p_I, p_E) \tag{3.19}$$

Let us define the differences of the first-order conditions as

$$\Delta_I = FOC_I^{a=c} - FOC_I^{a_1} = c\frac{\partial q_E(p_I, p_E)}{\partial p_I} - a_1$$

$$\Delta_E = FOC_E^{a=c} - FOC_E^{a_2} = -c\frac{\partial q_E(p_I, p_E)}{\partial p_E} + a_2$$

Notice that $\frac{\partial q_E(p_I,p_E)}{\partial p_I} > 0$ as well as $\frac{\partial q_E(p_I,p_E)}{\partial p_E} < 0$. In equilibrium $a_1 \leq 0$ and $a_2 \geq 0$. For $c \geq 0$, we have $\Delta_I > 0$ and $\Delta_E > 0$. Figure 3.4 illustrates the result qualitatively[19]. Consider the curve $BR_I^{a=c}2$. This is I's best response curve if $a = c$, the Bertrand duopoly. $BR_I^{a_1=a_1^*}$ depicts I's best response curve for Parapricing. Because $\Delta_I(a = c, a_1 = a_1^*) > 0$, the best response curve is shifted to the left.

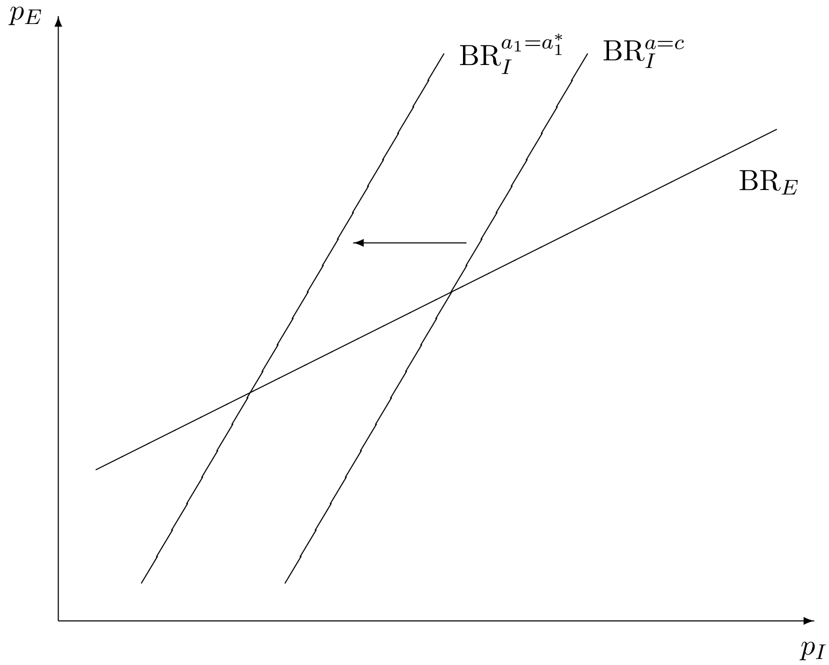

Figure 3.4: Changes in First order conditions

Hence under Parapricing retail prices necessarily have to be smaller than with (positive) per-unit charge.

3.5.2 Ramsey Pricing

It has been widely recognized that network industries involve high fixed cost as compared to the marginal cost of producing an additional unit of the network good. It is for that reason, that results have to compared to Ramsey prices instead of first best marginal cost

[19]The assumption of symmetric linear demands puts restrictions on the shape of the best response curve. In particular, they ensure linearity of best response curves and equal slopes for all cases.

prices. This subsection compares equilibrium prices under Parapricing with the corresponding Ramsey outcomes.

The representative consumer's utility function is maximized subject to the industry break even condition. More formally for the case of the linear demand specification this is

$$\max_{p_I,p_E} \; -\tfrac{1}{2(1-\sigma^2)}(q_I^2+q_E^2)-\tfrac{\sigma}{(1-\sigma^2)}q_Iq_E+\tfrac{1}{1-\sigma}(q_I+q_E)-p_Iq_I-p_Eq_E \qquad (3.20)$$
$$s.t.\; \Pi_I+\Pi_E=0$$

The following lemma describes the solution to the Ramsey programm.

Lemma 3.1. *There exist symmetric Ramsey prices $\{p_I^R, p_E^R\} \in \mathbb{R}_+^2$ whenever the following condition is met:*

$$\frac{\Big(1-2c(1-\sigma)\Big)^2}{2(1-\sigma}) \geq F_I+F_E$$

p_I^R and p_E^R are both increasing in marginal cost c and the sum of the fixed cost F_I+F_E.

Proof. See appendix.

The condition states, that Ramsey prices exist, whenever the multiproduct monopoly profit is greater than the sum of industry fixed cost. Given the existence of p_I^R and p_E^R we are able to state the following proposition:

Proposition 3.3. *Suppose that*

$$p_i^*(\hat{c},\hat{\sigma},\hat{F})-p_i^R(\hat{c},\hat{\sigma},\hat{F})=0$$

with $F=F_I+F_E$. Then

i. for all $c=\hat{c}, F=\hat{F}$ and $\sigma>\hat{\sigma} \Rightarrow p_i^*(\hat{c},\sigma,\hat{F})-p_i^R(\hat{c},\sigma,\hat{F})>0$

ii. for all $\sigma=\hat{\sigma}, F=\hat{F}$ and $c>\hat{c} \Rightarrow p_i^*(c,\hat{\sigma},\hat{F})-p_i^R(c,\hat{\sigma},\hat{F})<0$

iii. for all $c=\hat{c}, \sigma=\hat{\sigma}$ and $F>\hat{F} \Rightarrow p_i^*(\hat{c},\hat{\sigma},F)-p_i^R(\hat{c},\hat{\sigma},F)<0$

Proof. See appendix.

Proposition 3.3 is a central finding of the paper. It states that there exists a set of parameters, for which equilibrium prices under Parapricing are greater or equal to the optimal Ramsey prices found in lemma 3.1. For marginal cost low enough, Ramsey prices are smaller than Parapricing equilibrium prices. The higher the degree of substitutability,

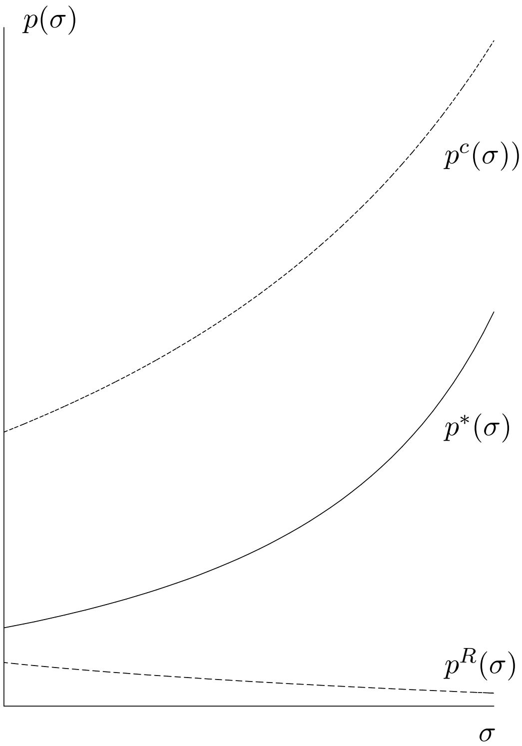

Figure 3.5: Prices

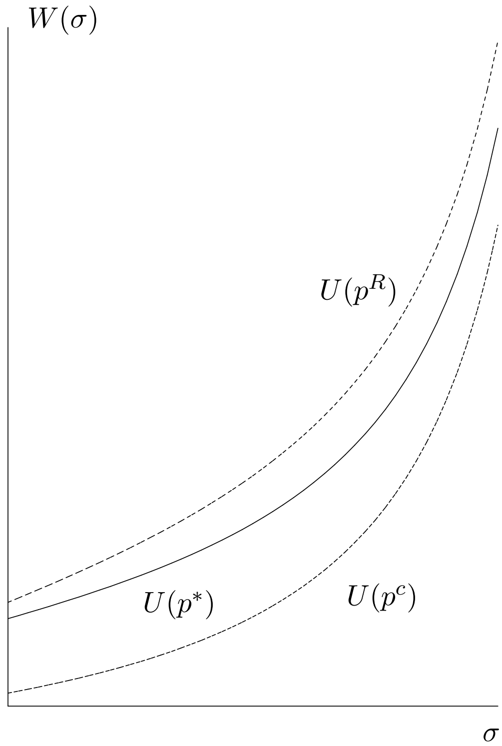

Figure 3.6: Welfare

the more likely we end up in an equilibrium with $p_i^R \leq p_i^*$. Lastly, whenever fixed cost are small enough, regulated equilibrium prices tend to be larger than Ramsey prices.

Together with the results of proposition 3.2 we are able to give an overall welfare ordering of the all three scenarios considered in the paper. The result is summarized in the proposition below.

Proposition 3.4. *Suppose that either $c \leq \hat{c}$ or $\sigma \geq s\hat{igm}a$ or $F \leq \hat{F}$, where $\hat{c}$, $\hat{\sigma}$ and $\hat{F}$ are defined as in proposition 3.3. Then the following inequalities hold:*

i. $p_i^R \leq p_i^* < p_i^a(c) \ \forall \ i = I, E$

ii. $U\big(q_E(p_I^R, p_E^R), q_I(p_I^R, p_E^R)\big) \geq U\big(q_E(p_I^*, p_E^*), q_I(p_I^*, p_E^*)\big)$

$> U\big(q_E(p_I^a(c), p_E^a(c)), q_I(p_I^a(c), p_E^a(c))\big)$

Proof. See appendix.

Figures 3.5 and 3.6 illustrate proposition 4.3 for variable degrees of product substitutability σ. Note that c, F_I and F_E are chosen such that σ^* is actually negative. Figure 3.5 plots the optimal Ramsey price p^R and equilibrium prices p^* and p^c under Parapricing and cost-based access pricing respectively. For all $\sigma \in]0, 1[$, Parapricing yields lower prices than cost -based access pricing.

This translates immediately into figure 3.6, which illustrates the second part of proposition 3.4. Consumer surplus is highest for Ramsey pricing. However Parapricing outperforms cost based pricing also in terms of welfare.

3.6 Discussion

Having stated the results, we want to discuss some of the assumptions of our model and extend the analysis.

■ **Differentiated Products:** With the Internet gaining more and more importance in today's business life, we believe that this is a necessary assumption to make. Not only "old fashioned" voice services use network capacity, but also content driven networks have to have access to physical networks in order to serve their customers.

Large and medium size enterprises buy network solutions. These can be LANs, company networks or simple telephone lines. In almost all cases, the contract between the telecommunication provider and the customer includes a range of products together with special agreements such as maintenance or customer service. In most countries these services are provided by the network proprietary and large resellers[20] of capacity. Differentiation simply stems from side agreements or the offered bundle in itself.

Online services such as video on demand, life-streaming and news services are provided by incumbents such as Deutsche Telekom in Germany. Original ISPs or competitors such as Skype and Yahoo that provide similar services buy connectivity from Deutsche Telekom. The differentiating aspects between the firms' offers are clearly contentwise[21]. Different providers specialize on different topics, so that a differentiated demand approach seems to be a good way to model the industry.

In general, network services, especially in the telecommunications industry, are being

[20]For example in Germany, T-Systems (a subsidiary of DTAG) and BT Ignite are direct competitors in the market for large and medium sized business customers. BT leases network capacity from DTAG, which and competes in the downstream market.

[21]Deutsche Telekom for example bought the rights to broadcast exclusively the Bundesliga, the first Division ins German Soccer, over the Internet. The UEFA broadcasts European Championsleague matches over the Internet. It goes without saying that the UEFA does not own a physical network to transmit the data.

sold as bundles. Even in the market for private customers, the different bundles that one could choose from are plentiful which satisfies the assumptions of differentiated goods environment.

■ **Linear demands:** Since most of the analysis is carried out using a specific linear demand schedule, the generality of the results is naturally very limited. However there are several reasons for which we think the procedure is justified.

The nature of the problem requires a workaround in order to derive any results at all. Armstrong, Doyle, and Vickers (1996) and Laffont and Tirole (1994) use a competitive fringe model that makes the analysis easier. For reasons laid out above, we think a model where both firms have market power is more appropriate.

The idea of shifting of the best response curves is a general result. It remains valid with any kind of demand curve. In simulations with logit demands, the results of the paper, in particular propositions 3.1 and 3.4 remain valid. This indicates a robustness of the results obtained with the linear demands.

■ **Selection of entrant(s):** In the present paper we restricted the analysis to a market structure with one entrant and an incumbent. Two questions arise immediately: how can firms commit to playing this game and what changes when there are several entrants?

Let us start by comparing the equilibrium profits of I and E:

$$\Pi_I(p_I^*(a_1^*, a_2^*), p_E^*(a_1^*, a_2^*)) - Pi_E(p_I^*(a_1^*, a_2^*), p_E^*(a_1^*, a_2^*)) = -2c$$

This stems from the fact that

$$a_1^* p_I^* + a_2^* p_E^* - cq_E(p_I^*(a_1^*, a_2^*), p_E^*(a_1^*, a_2^*)) = -c.$$

From an incentive point of view, we are faced with the problem that I has to make the necessary investments in the network, but the larger chunk of profits is made on E's side. Hence making firms commit to playing the regulatory game amounts to making the incumbent commit to playing the game. To circumvent this problem, we propose to auction off a license, which allows a firm to be the entrant and play the game. Given the appropriate auction design we expect firms to bid there expected profit, which is nothing but $\Pi_E(p_I^*(a_1^*, a_2^*), p_E^*(a_1^*, a_2^*))$. The auction revenues are then redistributed to the incumbent, such that the entrant hits his zero profit condition.

The mechanism is originally designed for two firms, of which one is the incumbent. However the game can extended to n entrants. Using a mechanism of the firm

$$A_I(p_I, p_{-I}) = (\sum_i^n b_i)p_I + \sum_i^n a_i p_i \tag{3.21}$$

$$A_i(p_I, p_i) = b_i p_I + a_i p_i \tag{3.22}$$

where $i = 1..n$, $p_{-I} = (p_1..p_n)$, $b_i \leq 0$ and $a_i \geq 0$ for all $i = 1..n$. Simulations for n=3 and linear demands suggest that the results obtained for a two player game persist. Equilibrium prices for the mechanism in (3.21) and (3.22) are again lower than those in a triopoly with access prices at marginal cost level. Notice that the mechanism exhibits the same features as the mechanism with one entrant. By picking his parameters the incumbent shifts the entrants' best response curves inwards, therefore lowering their prices and boost his own profits. The reverse is true for the entrants. By setting their parameters, they shift I's best response curve inwards and boost their own profits.

3.7 Conclusion

The present paper proposes a new approach to the regulation of interconnection. On realizing that using a per-unit access charge is nothing but a contract on retail prices, we employ a mechanism that takes this feature into account.

The mechanism puts both firms, the incumbent and the entrant into a game situation, where they non-cooperatively determine the entrant's access payment. This is a major difference to the existing literature on one-way interconnection, where the incumbent alone is proposing an access price. This contributes to the efficiency of the mechanism since it is less demanding in terms of information.

The resulting equilibrium prices are derived by employing a specific linear demand specification. However, simulations suggest that they are robust with respect to other demand schedules as well. In particular we find that under fairly reasonable assumptions such as, low marginal cost and products being close enough substitutes, an ordering prevails where equilibrium Ramsey prices are lowest, prices under an $\{a_1, a_2\}$-mechanism are slightly higher or equal and duopoly prices, i.e. the specific access price regulated at cost, are highest.

Analogously, welfare is maximal in the Ramsey program, $\{a_1, a_2\}$-mechanism welfare is second and access prices regulated at cost least favorable.

Since the linear model is very specific, natural extensions involve a general treatment of the proposed mechanism and the like, as well as extending the model to situations where n entrants are active in the market.

Another natural application of our proposal are models of two-way interconnection in the spirit of Armstrong (1998), Laffont et al. (1998a,b).

3.A Appendix

Proof of Proposition 3.1. In order to show the result, we derive best response functions for given parameter values in the second stage. We then turn to the analysis of the first stage and show, that proposition 3.1 characterizes indeed a sub game perfect equilibrium.

Before turning to firms' best response curves, note that profit functions have are defined piecewise due to the demand specification. In particular,

$$\Pi_I = \begin{cases} \Pi_I^M & \text{for} \quad 0 \leq p_I \leq \frac{p_E - 1}{\sigma} \\ \Pi_I^D & \text{for} \quad \frac{p_E - 1}{\sigma} < p_I < 1 + \sigma p_E \\ A(p_I, p_E) & \text{for} \quad 1 + \sigma p_E \leq p_I \end{cases} \tag{A.3.1}$$

$$\Pi_E = \begin{cases} \Pi_E^M & \text{for} \quad 0 \leq p_E \leq \frac{p_I - 1}{\sigma} \\ \Pi_E^D & \text{for} \quad \frac{p_I - 1}{\sigma} < p_E < 1 + \sigma p_I \\ -A(p_I, p_E) & \text{for} \quad 1 + \sigma p_I \leq p_E \end{cases} \tag{A.3.2}$$

where

$$\begin{aligned} \Pi_I^D &= (p_I - 2c) q_I(p_I, p_E) - c q_E(p_I, p_E) + a_1 p_I + a_2 p_E \\ \Pi_E^D &= (p_E - c) q_E(p_I, p_E) - a_1 p_I - a_2 p_E. \end{aligned}$$

whenever both firms face positive demand and

$$\begin{aligned} \Pi_I^M &= (p_I - 2c)(1 + \sigma)\big(1 - (1 - \sigma) p_I\big) + a_1 p_I + a_2 p_E \\ \Pi_E^M &= (p_E - c)(1 + \sigma)\big(1 - (1 - \sigma) p_E\big) - a_1 p_I - a_2 p_E \end{aligned}$$

whenever prices are such that either firm is driven out of the market.

2nd Stage

The equilibrium of the pricing game has to be analyzed given the choice of a_1 and a_2 in the previous period of the game. That means, we need an equilibrium choice of (p_I, p_E) for every possible combination of $a_1 \leq 0$ and $a_2 \geq 0$.

Before deriving best response functions, let us introduce important cut off values and clarify notation for the reminder. The first thing that is worth mentioning concerns the functions Π_I^M, Π_E^M, Π_I^D and Π_E^D. Note that all these functions are quadratic and concave in their respective prices, i.e. E's profits are concave in p_E whereas I's profits are concave in p_I. It follows that each exhibit a maximum in the corresponding price. They are given by[22]

$$p_I^M = \frac{1+\sigma+a_1}{2(1-\sigma^2)} + c \tag{A.3.3}$$

$$p_E^M = \frac{1+\sigma-a_2}{2(1-\sigma^2)} + \frac{c}{2} \tag{A.3.4}$$

$$p_I^D = \frac{1}{2}(1+a_1+\sigma p_E - c\sigma) + c \tag{A.3.5}$$

$$p_E^D = \frac{1}{2}(1-a_2+\sigma p_I + c). \tag{A.3.6}$$

Whether these prices are in a firm's feasible set depends crucially on the pricing decision of the opponent firm. As discussed above, the price spread determines which demand the firm faces, hence it also determines, of which shape the profit functions are. The other determinants of these maxima are the coefficients a_1 and a_2. It makes sense to indicate this by using the notation $p_I^M(a_1)$, $p_E^M(a_2)$, $p_I^D(a_1, p_E)$ and $p_E^D(a_2, p_I)$.

The cut off prices for which a firm switches from monopolistic to duopolistic behavior are also of great importance to the analysis of the optimal pricing strategy. For them will use the notation

$$\begin{aligned} p_I^S(p_E) &= \frac{p_E - 1}{\sigma} \\ p_E^S(p_I) &= \frac{p_I - 1}{\sigma}. \end{aligned}$$

At these points the profit functions are non-differentiable, since for $p_i \leq p_i^S$ firm i has to deal with Π_i^M, whereas for $p_i > p_i^S$ Π_i^D is valid.

Depending on the parameters a_1 and a_2, given prices p_I and p_E and the shape of the different parts of (A.3.1) and (A.3.2), overall profits can be single-peaked or double-peaked[23]. For deriving the best response functions it is most instructive to look at characteristic intervals of a_1 and a_2. Given these intervals, we identify the best response curve by varying the opponents price.

In order to identify these intervals, we have to introduce two more prices for each firm.

[22]Note that super- and subscripts are in line with the notation of the respective profit functions.

[23]This follows from the quadratic and concave nature of (A.3.1) and (A.3.2).

1. $\lambda_E = p_E | p_I^S(p_E) = p_I^D(p_E, a_1)$. Notice that for $p_E \leq \lambda_E$ implies $p_I^D(p_E, a_1) \geq p_I^S(p_E)$. In line with that definition, $\lambda_I = p_I | p_E^S(p_I) = p_E^D(p_I, a_2)$.

2. $\tau_E = p_E | p_I^S(p_E) = p_I^M(a_1)$. Since $p_I^S(p_E)$ is increasing in p_E, $p_E \leq \tau_E$ implies $p_I^S(p_E) \geq p_I^M(a_1)$. Analogously $\tau_I = p_I | p_E^S(p_I) = p_E^M(a_2)$.

In our model, these values turn out to be

$$\begin{aligned}
\lambda_E(a_1) &= \frac{2 + \sigma(1 + a_1 + 2c - c\sigma)}{2 - \sigma^2} \\
\lambda_I(a_2) &= \frac{2 + \sigma(1 + c - a_2)}{2 - \sigma^2} \\
\tau_E(a_1) &= \frac{2 + \sigma(1 + a_1 + 2c - \sigma - 2c\sigma^2)}{2(1 - \sigma^2)} \\
\tau_I(a_2) &= \frac{2 + \sigma(1 + c - a_2 - \sigma - c\sigma^2)}{2(1 - \sigma^2)}.
\end{aligned}$$

The arguments of λ's and τ's emphasize the dependence on parameter values a_1 and a_2. It is easy to verify, that $\lambda_E(a_1)$ and $\tau_E(a_1)$ are increasing in a_1, whereas $\lambda_I(a_2)$ and $\tau_I(a_2)$ are decreasing in a_2. Furthermore, $\frac{\partial \lambda_E}{\partial a_1} < \frac{\partial \tau_E}{\partial a_1}$ and $\frac{\partial \lambda_I}{\partial a_2} > \frac{\partial \tau_I}{\partial a_2}$.

The relationship of (A.3.3)-(A.3.6), $\lambda_E(a_1)$, $\lambda_I(a_2)$, $\tau_E(a_1)$ and $\tau_I(a_2)$ depends on a_1 and a_2. To keep the notation as simple as possible, we define the following threshold values upfront:

$$\begin{aligned}
\alpha_1^1 &= c\sigma - \sigma - 1 - 2c & \alpha_2^1 &= 1 + \sigma + c \\
\alpha_1^2 &= 2c\sigma^2 - \sigma - 1 - 2c & \alpha_2^2 &= 1 + \sigma + c - c\sigma^2 \\
\alpha_1^3 &= -(2c(1-\sigma)^2 + \sigma)\frac{1+\sigma}{\sigma} & \alpha_2^3 &= (1 - c(1 - \sigma))(1 + \sigma) \\
\alpha_1^4 &= -1 - 2c + c\sigma & \alpha_2^4 &= 1 + c
\end{aligned}$$

Subscripts indicate the respective parameter, i.e. α_1^i is a specific value for a_1, the parameter that firm E decides upon. For future reference it has to be noted that $\alpha_1^1 < \alpha_1^2 < \alpha_1^3 < \alpha_1^4 < 0$ and $0 < \alpha_2^5 < \alpha_2^4 < \alpha_2^3 < \alpha_2^2 < \alpha_2^1$ given our parameter restrictions of $0 \leq c < \frac{1}{2} < \sigma \leq 1$. Hence, $\{a_1, a_2\}$-space is subdivided into mutually exclusive sets.

It is easy to verify that for $a_1 \leq \alpha_1^1$ neither $p_I^D(a_1, p_E)$ nor $p_I^M(a_1)$ are feasible for $p_E \geq 0$. For $a_2 \geq \alpha_2^1$ neither $p_E^D(a_2, p_I)$ nor $p_E^M(a_2)$ are in E's feasible set for $p_I \geq 0$..

Whenever $\alpha_1^1 \leq a_1 \leq \alpha_1^2$, only $p_I^D(a_1, p_E)$ is feasible for some $p_E \geq 0$. Likewise, $\alpha_2^2 \leq a_2 \leq \alpha_2^1$ implies that only $p_E^D(a_2, p_I)$ is feasible for some $p_I \geq 0$. For $\alpha_1^2 \leq a_1 \leq \alpha_1^3$, both $p_I^M(a_1)$ and $p_I^D(a_1, p_E)$ are feasible for some $p_E \geq 0$. Again, for $\alpha_2^3 \leq a_2 \leq \alpha_2^2$ both $p_I^M(a_1)$ and $p_I^D(a_1, p_E)$ are part of Π_E for some $p_I \geq 0$. Furthermore it is easy to verify that $a_1 \leq \alpha_1^3$ implies that $\lambda_E(a_1) \geq \tau_E(a_1)$ and $a_2 \geq \alpha_2^3$ implies that $\lambda_I(a_2) \geq \tau_I(a_2)$. But $|a_i| \geq |\alpha_i^3|$ implies that $\lambda_j(a_i) \geq \tau_j(a_i)$. Finally for $|a_i| \geq |\alpha_i^4|$, $p_i^D(a_i, 0) \leq 0$, whereas for $|a_i| < |\alpha_i^4|$, $p_i^D(a_i, 0) > 0$[24].

The relationship between λ's and τ's determines the shape of the profit functions in (A.3.1) and (A.3.2) for different values of p_E and p_I. In general, there are two possibilities: either we have $\lambda_i > \tau_i$ or $\lambda_i > \tau_i$. Therefore α_1^3 and α_2^3 are important benchmarks to look at, since $\lambda_E(\alpha_1^3) = \tau_E(\alpha_1^3)$. Because of the slope conditions and the linearity of λ's and τ's in a_1 and a_2, $\lambda_E(a_1) > \tau_E(a_1)$ for $a_1 < \alpha_1^3$ and $a_2 > \alpha_2^3$. By the definition of λ_i and τ_i, we know that for $p_i \leq \tau_i(a_j)$, $i = I, E$, Π_j is a single-peaked function, with the global maximum at $p_j^D(a_j, p_i)$. Whenever $\tau_i(a_j) < p_i < \lambda_i(a_j)$, the profit of firm j is bi-modal. The two local maxima are $p_j^M(a_j)$ and $p_j^D(a_j, p_i)$ where we have the the relation $p_j^M(a_j) < p_j^D(a_j, p_i)$. For $p_i \geq \lambda_i(a_j)$, Π_j is again single-peaked with the global maximum occurring at $p_j^M(a_j)$.

Let us now turn to the situation where $a_1 > \alpha_1^3$ and $a_2 < \alpha_2^3$, that is $\lambda_i(a_j) < \tau_i(a_j)$. This implies that for $p_i \leq \lambda_i(a_j)$, the only viable maximum of Π_j is $p_j^* = p_j(a_j, p_i)$. For $\lambda_i(a_j) < p_i < \tau_i(a_j)$, the function Π_j is again single-peaked. However neither $p_j^M(a_j)$ nor $p_j^D(a_j, p_i)$ are in the feasible set of firm j. Consider $p_j^S(p_i)$, the price at which the change in regimes from Π_j^M to Π_j^D occurs. To the right ($p_j > p_j^S(p_i)$) Π_j^D is valid. Its maximum occurs at $p_j^D(a_j, p_i) < p_j^S(p_i)$, hence Π_j is decreasing for $p_j > p_j^S(p_j)$. To the left ($p_j < p_j^S(p_i)$) Π_j^M is valid, for which the maximum occurs at $p_j^M(a_j) > p_j^S(a_j)$. Hence Π_j is increasing up to $p_j^S(p_i)$. This implies that the global maximum occurs at $p_j^S(p_i)$. For $p_i \geq \tau_i(a_j)$, j's profit is again single-peaked with $p_j^* = p_j^M(a_j)$. Equipped with this information, we can now turn to the derivation of I's and E's best response curves, $\hat{p}_(a_1, p_E)$ and $\hat{p}_E(a_2, p_I)$ respectively.

[24]Notice that subscript i on prices refers to a firm, whereas on parameters it refers to a number. However 1 is meant to correspond with firm I and 2 with E.

I's Best Response Curves

To account for different shapes of I's profit, we proceed by case based analysis of the best response function. We do this by fixing a_1 to be in different intervals. Note that this makes I's profit a function of p_E alone. Therefore, within the different cases, p_E is varied.

i) $a_1 \leq \alpha_1^1$: Since $\alpha_1^1 < \alpha_1^3$ we know that $\tau_E(a_1) < \lambda_E(a_1)$. We also know that by means of the regulatory mechanism, prices are required to be non-negative. This puts an additional restriction on $p_I^D(a_1, p_E)$ and $p_I^M(a_1)$.

   Let us start with $0 \leq p_E \leq 1$. Whenever this is the case, we know that $p_I^S(p_E) \leq 0$. This implies that I has no opportunity to set p_I such that his opponent is driven out of the market. In other words, Π_I^D is the relevant branch of the profit function for $p_I \geq 0$. Notice that $\lambda_E(\alpha_1^1) = 1$. This is to say, that $p_I^D(\alpha_1^1, 1) = p_I^S(1)$=0. Because $\frac{\partial \lambda_E}{\partial a_1} > 0$, we have $\lambda_E(a_1) < 1$ for all $a_1 < \alpha_1^1$. Hence for $0 \leq p_E \leq 1$, Π_I^D is a decreasing function for $p_I \geq 0$. This in turn implies that the best response to any $p_E \in [0, 1]$ is setting $\hat{p}_I(a_1, p_E) = 0$.

   Whenever $p_E > 1$, we also know that $p_E > \lambda_E(a_1)$. Hence Π_I is single-peaked with the maximum occurring at $p_I^M(a_1)$. It is easy to verify that for $a_1 \leq \alpha_1^1$, this is always negative, meaning that it is never in the feasible set of firm I. Since we know that for $p_E \geq \lambda_E(a_I)$, $p_I^M(a_1)$ is the only maximum of Π_I, which is also non-increasing beyond $p_I^M(a_1)$, we can conclude that again the optimal pricing strategy for I is picking $\hat{p}_I(a_1, p_E) = 0$.

ii) $\alpha_1^1 < a_1 \leq \alpha_1^2$: The parameter restrictions imply special relationships. First of all, it is important to realize, that for $a_1 \leq \alpha_1^2$, $p_I^M(a_1) \leq 0$. Secondly we know that $\lambda_E(a_1) > 1$ for $a_1 \in [\alpha_1^1, \alpha_1^2]$. Furthermore, since $\alpha_1^2 < \alpha_1^3$, we know that $\tau_E(a_1) < \lambda_E(a_1)$.

   Let us again start with $0 \leq p_E \leq 1$. $p_I^S(p_E) \leq 0$, which means that for $p_I \geq 0$, $\Pi_I = \Pi_i^D$. $\lambda_E(a_1) > 1$ implies that there has to be an interval $[\eta_0(a_1, 1]$, where $p_I^D(a_1, p_E) \geq 0$. This is the global maximum of Π_I^D and therefore the best response of I to $p_E \in [\eta_0(a_1), 1]$. Because $p_I^D(a_1, p_E)$ is increasing in p_E, it is less than zero for $p_E \in [0, \eta_0(a_1)[$.

Now consider the case of $p_E \in]1, \lambda_E(a_1)[$. Π_I is a bimodal function, with a local maximum $p_I^M(a_1) < 0$ and $p_I^D(a_1, p_E) > 0$. For $p_I \leq p_I^S(p_E)$, $\Pi_I = \Pi_I^M$ which is decreasing in p_I. For $p_I > p_I^S(p_E)$, $\Pi_I = \Pi_I^D$, which is maximized at $p_I^D(a_1, p_E) > p_I^S(p_E)$. This means that for $0 \leq p_I \leq p_I^S(p_E)$, Π_I is decreasing, whereas for $p_I^S(p_E) < p_I \leq p_I^D(a_1, p_E)$ it is increasing. The candidates for a best response are either $p_I = 0$ or $p_I = p_I^D(a_1, p_E)$ and whenever $\Pi_I^M(p_I = 0) > \Pi_I^D(p_I = p_I^D(a_1, p_E))$, I picks the former and vice versa. Let us now consider the function

$$\Delta_0(p_E, a_1) = \Pi_I^M(p_I = 0, p_E) - \Pi_I^D(p_I = p_I^D(a_1, p_E), p_E). \tag{A.3.7}$$

Notice that for $p_E = 1$, i.e. $p_I^S(1) = 0$, $\Delta_0(p_E, a_1) < 0$ because Π_I is a continuous function. Furthermore at $p_E = \lambda_E(a_1)$, $\Delta_0(p_E, a_1)$ is positive, by the same argument. Because of its components, $\Delta_0(p_E, a_1)$ is concave and quadratic in p_E, hence exhibits at most two roots that solve $\Delta_0(p_E, a_1) = 0$. Notice that because of the concavity condition, $\Delta_0(p_E, a_1) > 0$ in between these two roots. This also implies that we have to look for the smaller of these two roots, because we know that in the p_E-interval that we are looking at, $\Delta_0(p_E, a_1)$ moves from negative to positive. Let us call this value $\eta_1(a_1)$. It is given by

$$\eta_1(a_1) = \frac{-2c - \sigma(c\sigma - 1 - a_1 + 2c) + 2\sqrt{c(2\sigma - 1)(\sigma(c\sigma - \sigma - 1 - a_1) - c)}}{\sigma^2}$$

To show that $\eta_1(a_1)$ is in fact a real number, notice that the term in the square root is decreasing in a_1. Hence it reaches its minimal value at α_1^2, the upper bound of the interval. But this is given by

$$c(2\sigma - 1)(\sigma(c\sigma - \sigma - 1 - \alpha_1^2) - c) = c^2(2\sigma - 1)(1 - \sigma^2) > 0.$$

Furthermore $1 < \eta_1(a_1) < \lambda_E(a_1)$ because $\Delta_0(a_1, 1) < 0$ and $\Delta_0(a_1, \lambda_E(a_1)) > 0$. Notice that because of the second inequality, the other root of $\Delta_0(p_E, a_1) = 0$ becomes irrelevant since it is larger than $\lambda_E(a_1)$. In that case, $p_I^D(a_1, p_E)$ is not part of I's feasible set anymore. Hence for $p_E \geq \eta_1(a_1)$, I's best response turns out to be $\hat{p}_I(a_1, p_E) = 0$. For The interval $[\eta_1(a_1), \lambda_E(a_1)]$, we found that $\hat{p}_I(a_1, p_E)0$ must be I's optimal strategy. For $p_E > \lambda_E(a_1)$ Π_I is decreasing for $p_I \geq 0$, hence again we have $\hat{p}_I(a_1, p_E) = 0$.

Formally the best response is given by

$$\hat{p}_I(a_1, p_E) = \begin{cases} 0 & for \quad 0 \leq p_E < \eta_0(a_1) \\ p_I^D & for \quad \eta_0(a_1) \leq p_E < \eta_1(a_1) \\ 0 & for \quad \eta_1(a_1) \leq p_E \end{cases} \tag{A.3.8}$$

where

$$\eta_0(a_1) = \frac{c\sigma - 1 - a_1 - 2c}{\sigma}.$$

iii) $\alpha_1^2 < a_1 \leq \alpha_1^3$: Whenever a_1 satisfies the conditions above, the analysis for $p_E \in [0, 1]$ remains as in case ii). $p_I^S(p_E) \leq 0$ is always the case for $o \leq p_E \leq 1$, which implies that $\Pi_I = \Pi_I^D$ and $p_I^D(a_1, p_E) \geq 0$ remains the maximum for $\eta_0(a_1) \leq p_E \leq 1$. However notice that $\eta_0(a_1)$ is decreasing in a_1. Nevertheless it is still positive because $\eta_0(\alpha_1^4) = 0$ and $\alpha_1^4 > \alpha_1^3$.

$a_1 \geq \alpha_1^2$ implies $p_I^M(a_1) \geq 0$, which means that the analysis changes for $p_E > 1$. $\tau_E(a_1)$, the value at for which $p_I^S(p_E) = p_I^M$ is now greater than one. This has to be the case because $p_I^S(p_E) > 0$ only for values of $p_E > 1$. But for $a_1 \leq \alpha_1^3$ we know that $\lambda_E(a_1) \geq \tau_E(a_1) > 0$. Since we already know that for $p_E \leq \tau_E(a_1)$ the function Π_I is single peaked with its maximum occurring at $p_I = p_I^D(a_1, p_E)$, we can conclude that this is I's optimal choice of p_I up to $\tau_E(a_1)$.

For $p_E \in [\tau_E(a_1, \lambda_E(a_1)]$ I's profit function exhibits two local maxima, which are both positive, namely $p_I^M(a_1)$ and $p_I^D(a_1, p_E)$. In line with the discussion in ii) let us define

$$\Delta_1(p_E, a_1) = \Pi_i^M(p_I^m(a_1), p_E) - \Pi_I^D(p_I^D(a_1, p_E), p_E).$$

Whenever $\Delta_1(p_E, a_1) < 0$ I chooses $\hat{p}_I(a_1, p_E) = p_I^D(a_1, p_E)$, whenever $\Delta_1(p_E, a_1) > 0$, $\hat{p}_I(a_1, p_E) = p_I^M(a_1)$. Again $\Delta_1(p_E, a_1)$is quadratic and concave in p_E, which means that we find at most two p_E that satisfy $\Delta_1(p_E, a1) = 0$. It is important to realize that $\Delta_1(\tau_E(a_1), a_1) \leq 0$ whereas $\Delta_1(\lambda_E(a_1), a_1) > 0$. This simply follows from continuity and the characteristics if Π_I at $\lambda_E(a_1)$ and $\tau_E(a_1)$. It implies that we are looking for the smaller of the two roots, because then $\Delta_1(p_E, a_1)$ changes from negative to positive which is due to its quadratic and concave form. It is easy to verify that this root is given by

$$\eta_2 = \frac{c\sigma^2 + 2c\sigma - \sigma - a_1\sigma - 2c}{\sigma^2} - \frac{2c(1-\sigma)(1-\sigma^2) + \sigma(1+\sigma+a_1)}{\sigma^2\sqrt{(1-\sigma^2)}}.$$

When p_E is increased beyond η_2, Π_1 is still bi-modal. However as $\Delta_1 > 0$ $p_I^M(a_1)$ is the global maximum. This is actually true for the interval $[\eta_2, \tau_E(a_1)]$. At $\tau_E(a_1)$, $p_I^S(p_E) = p_I^D(a_1, p_E)$ is by definition satisfied. To the left of the cut-off point, we are on the decreasing branch of Π_I^M, exactly at $p_I^S(p_E)$ the maximum of Π_I^D occurs, hence for $p_I > p_I^S$ $\Pi_I = \Pi_I^D$ which is decreasing. This implies that at $p_E = \tau_E(a_1)$ the function Π_I exhibits a point of terrace.

As we already saw above, for $p_E > \lambda_E(a_1)$ I's profit is single-peaked with a global maximum at $p_I^M(a_1) > 0$. We can now state the best response of I for the whole interval as given by

$$\hat{p}_I(a_1, p_E) = \begin{cases} 0 & for \quad 0 \leq p_E < \eta_0(a_1) \\ p_I^D & for \quad \eta_0(a_1) \leq p_E < \eta_2(a_1) \\ p_I^M & for \quad \eta_2(a_1) \leq p_E \end{cases} \tag{A.3.9}$$

iv) $\alpha_1^3 < a_1 \leq \alpha_1^4$: The first inequality implies that $1 < \lambda_E(a_1) < \tau_E(a_1)$ and the second insures that $\eta_0(a_1) \geq 0$, hence stetting $\hat{p}_I(a_1, p_E) = 0$ is a best response to $0 \leq p_E \leq \eta_0(a_1)$.

I's best response for $p_E \in [0,1]$ is by now familiar. Since $p_I^S \leq 0$, we have $\Pi_I = \Pi_I^D$ for $p_I \geq 0$. we also know that for $p_E \leq \eta_0$, $p_I^D(a_1, p_E) \leq 0$. Hence for $p_E > \eta_0(a_1)$, $\hat{p}_I(a_1, p_E) = p_I^D(a_1, p_E)$.

Now consider the case of $p_E > 1$. We know that $1 < \lambda_E(a_1) < \tau_E(a_1)$. This gives us a very clear picture of how Π_I looks like, as we increase p_E. For $1 < p_E \leq \lambda_E(a_1)$ I's profit peaks at $p_I = p_I^D(a_1, p_E)$. Qualitatively we can think of the graph of $\Pi_I(p_I)$ as follows: for $0 \leq p_I \leq p_I^S(p_E)$ we have $\Pi_I = \Pi_I^M$, which is increasing. $p_I^M(a_1)$ is not in I's feasible set until $p_E \geq \tau_E(a_1)$. For $p_I > p_I^S(p_E)$, $\Pi_I = \Pi_I^D$. Because $p_E \leq \lambda_E(a_1)$, this function peaks at $p_I^D(a_1, p_E) > p_I^S(p_E)$. Hence I's optimal pricing strategy for $1 < p_E \leq \lambda_E(a_1)$ is $\hat{p}_I(a_1, p_E) = p_I^D$.

For $\lambda_E(a_1) < p_E \leq \tau_E(a_1)$ the picture changes such that $p_I^D(a_1, p_E) < p_I^S(p_E)$. This simply says that neither $p_I^D(a_1, p_E)$ nor $p_I^M(a_1)$ are part of the feasible set anymore. However observe that Π_I exhibits a unique global maximum at $p_I^S(p_E)$, because to the left of this point $\Pi_I = \Pi_i^M$, which is increasing ($p_E \leq \tau_E(a_1)$). To the right $\Pi_I = \Pi_I^D$ is decreasing ($p_E > \lambda_E(a_1)$). This means that for any $p_E \in [\lambda_E(a_1), \tau_E(a_1)]$, $\hat{p}_I(a_1, p_E) = p_I^S(p_E)$.

The last inequality to analyze is $\tau_E(a_1) < p_E$, which implies that $p_I^M(a_1) < p_I^S$. Given what has been said so far, the situation is straightforward. As before, to the right of $p_I^S(p_E)$, the function decreasing. If I drives E out of the market, i.e. $p_I \leq p_I^S(p_E)$, $\Pi_I = \Pi_I^M$. Hence the global maximum is $\hat{p}_I(a_1, p_E) = p_I^M(a_1)$.

Summing up this case, we find that the the best response of I is given by

$$\hat{p}_I(a_1, p_E) = \begin{cases} 0 & for & 0 & \leq & p_E & < \eta_0(a_1) \\ p_I^D & for & \eta_0(a_1) & \leq & p_E & < \lambda_E(a_1) \\ p_I^S & for & \lambda_E(a_1) & \leq & p_E & < \tau_E(a_1) \\ p_I^M & for & \tau_E(a_1) & \leq & p_E & \end{cases} \tag{A.3.10}$$

v) $\alpha_1^4 < a_1 \leq 0$: This case can be analyzed very easily. Compared to case iv) nothing actually changes except that $\eta_0(a_1) < 0$. This implies that $p_I = 0$ is never a best response for I to any p_E. The rest of the analysis remains as laid down in iv). Hence the best response is given by

$$\hat{p}_I(a_1, p_E) = \begin{cases} p_I^D & for & 0 & \leq & p_E & < \lambda_E(a_1) \\ p_I^S & for & \lambda_E(a_1) & \leq & p_E & < \tau_E(a_1) \\ p_I^M & for & \tau_E(a_1) & \leq & p_E & \end{cases} \tag{A.3.11}$$

These five different cases characterize I's optimal choices of p_I for any $a_1 \in [-\infty, 0]$. It is important to notice, that any of the (A.3.8), (A.3.9), (A.3.10) or (A.3.11) depend on a_2. In the next section we present the same exercise for firm E. Just as in this section, we can identify five different intervals, conditioned on a_2.

E's Best Response Curves

The restriction on a_2 to be non-negative together with the four different cut off values $\alpha_2^1 - \alpha_2^4$ define five mutually exclusive sets in a_2-space. Hence we have to look at five different cases. Within each, a_2 is fixed, only p_I is varied from zero to infinity. Using this strategy we derive the best response function $\hat{p}_E(a_2, p_I)$ for every a_2-interval. Notice that the different sets of a_2 imply different shapes of profit functions, for $p_I \geq 0$. Characteristic plots of these profit functions are provided in appendix B.

i) $a_2 \geq \alpha_2^1$: For all $a_2 > \alpha_2^3$ we know that $\tau_I(a_2) < \lambda_I(a_2)$. Since $\alpha_2^1 > \alpha_2^3$, this is certainly satisfied here. Additionally we know that for $a_2 \geq \alpha_2^1$, $p_E^M(a_2)$ and $p_E^D(a_2, p_I)$ are never feasible strategies of E.

   Consider $0 \leq p_I \leq 1$. By the definition of Π_E and $p_E^S(p_I)$ we know that E does have no opportunity to drive I out of the market, since $p_E^S(p_I) \leq 0$. Hence $\Pi_E = \Pi_E^D$ is the relevant part of the E's profit for $p_E \geq 0$. Because $\lambda_I(\alpha_2^1) = 1$, the ordering $p_E^M(a_2) \leq p_E^S(p_I) \leq p_E^D(a_2, p_I) \leq 0$ is valid for $0 \leq p_I \leq \lambda_I(a_2) \leq 1$. Because of the non-negativity restriction on prices, neither $p_E^D(a_2, p_I)$ nor $p_E^M(a_2)$ are in E's feasible set. For $p_I \geq \lambda_I(a_2)$ $p_E^D(a_2, p_I) \leq p_E^S(p_I)$, hence it is not on the valid branch of Π_E. Notice that $p_E^M(a_2)$ does not depend on p_I and is always smaller than zero for $a_2 \in [\alpha_2^1, \infty]$. Hence although $p_E^S(p_I) > \tau_I(a_2)$, $p_E^M(a_2)$ is not feasible. Therefore, E's profit is always downward sloping for $p_E \geq 0$ and $p_I \in [0, 1]$ and the optimal price is $\hat{p}_E(a_2, p_I) = 0$

   For $p_I > 1$, the picture does not change. Recall that $p_E^M(a_2) \leq p_E^D(a_2, p_I) < p_E^S(p_I)$ and $p_E^S(p_I) > 0$. Hence Π_E is always decreasing for $p_E \geq 0$. Notice that for $0 \leq p_E \leq p_E^S(p_I)$, Π_E^M is valid. Since $p_E^M(a_2) < 0$, this part is also downward sloping. To sum up, we can see that for $a_2 \geq \alpha_2^1$, The best response of E to any p_I is $\hat{p}_E(a_2, p_I) = 0$.

ii) $\alpha_2^1 > a_2 \geq \alpha_2^2$: Recall that α_2^2 is defined such that for all $a_2 \geq \alpha_2^2$, $p_E^M(a_2) \leq 0$. Additionally since $\alpha_2^2 > \alpha_2^3$, $\lambda_I(a_2) > \tau_I(a_2)$.

   Consider $0 \leq p_I \leq 1$. Because $a_2 < \alpha_2^1$, $\lambda_I(a_2) > 1$. For $p_I = 1$, we have $0 = p_E^S < p_E^D(a_2, p_I)$, where the last inequality stems from the fact, that $\lambda_I(a_2) > 1$. Recall that $\alpha_2^2 < \alpha_2^4$, hence $p_E^D(a_2, 0) < 0$. Since $p_E^D(a_2, p_I)$ is increasing in p_I, there has to exist a $p_I = \theta_0(a_2) > 0$, for which $p_E^D(a_2, p_I) = 0$. Therefore, for $0 \le p_I \le \theta_0(a_2)$, the optimal choice of E is $\hat{p}_E(a_2, p_I) = 0$. For $\theta_0(a_2) < p_I \le 1$, $\hat{p}_E(a_2, p_I) = p_E^D(a_2, p_I)$.

For $]1, \lambda_I(a_2)]$, Π_E is bimodal, with the ordering $p_E^M \le 0 < p_E^S(p_I) \le p_E^D(a_2, p_I)$. Notice that

$$\frac{\partial \Pi_E}{\partial p_E}|_{p_E \in (0, p_E^S(p_I)]} = \frac{\partial \Pi_E^M}{\partial p_E}|_{p_E \in (0, p_E^S(p_I)]} < 0 \qquad \text{and}$$

$$\frac{\partial \Pi_E}{\partial p_E}|_{p_E \in [p_E^S(p_I), p_E^D(a_2, p_I)]} = \frac{\partial \Pi_E^D}{\partial p_E}|_{p_E \in [p_E^S(p_I), p_E^D(a_2, p_I)]} \ge 0,$$

because for $p_E \le p_E^S(p_I)$, $\Pi_E = \Pi_E^M$ which is maximized at $p_E^M(a_2) \le 0$ and for $p_E > p_E^S(p_I)$, $\Pi_E = \Pi_E^D$ which is maximized at $p_E^D(a_2, p_I) \ge p_E^S(p_I)$. This in turn says that the candidate equilibria are either $p_E = 0$ if $\Pi_E^M(p_I, p_E = 0) > \Pi_E^D(p_I, p_E = p_E^D(a_2, p_I))$ or $p_E = p_E^D(a_2, p_I)$ otherwise.

Now consider the function

$$\Gamma_0(a_2, p_I) = \Pi_E^M(p_I, p_E = 0) - \Pi_E^D(p_I, p_E = p_E^D(a_2, p_I)).$$

Because $\Pi_E^M(p_I, p_E^S(p_I)) = \Pi_E^D(p_I, p_E^S(p_I))$, we know that for $p_I = 1$, $\Gamma_0(a_2, p_I) < 0$. However since Π_E^M is decreasing in $p_E \ge 0$, we have $\Gamma_0(a_2, p_I) > 0$ at $p_I = \lambda_I(a_2)$. Recall that $\lambda_I(a_2)$ is the value of p_I, for which $p_E^D(a_2, p_I) = p_E^S(p_I)$. Hence, for $p_I > \lambda_I(a_2)$, $p_E^M(a_2) < 0 < p_E^D(a_2, p_I) < p_E^S(p_I)$. Because of its building blocks, $\Gamma_0(a_2, p_I)$ is concave and quadratic in p_I. Hence there are at most two roots in p_I satisfying $\Gamma_0(a_2, p_I) = 0$. Notice that we are looking for the smaller root of the two. From the shape of $\Gamma_0(a_2, p_I)$ it is easy to verify that we have to move from $\Gamma_0(a_2, p_I) < 0$ to $\Gamma_0(a_2, p_I) > 0$. This is indeed the case for the p_I-interval we interested in. Solving $\Gamma_0(a_2, p_I) = 0$ and calling the smaller root $\theta_1(a_2)$, we get

$$\theta_1(a_2) = \frac{c - 1 + a_2 - 2\sqrt{(c(a_2 - 1 - \sigma)}}{\sigma}.$$

For $a_2 \in [\alpha_2^2, \alpha_2^1]$, $\theta_1(a_2)$ is indeed a real number, because $\alpha_2^2 > 1 + \sigma$. Notice that by definition, $\theta_1(a_2) > 1$. Furthermore the second root of $\Gamma_0(a_2, p_I) = 0$ is irrelevant because it is larger than $\lambda_I(a_2)$. In between the two roots, $\Gamma_0(a_2, p_I) > 0$, which is certainly the case for $\Gamma_0(a_2, p_I = \lambda_I(a_2))$.

For $p_I > \lambda_I(a_2)$, $p_I^M(a_2) \leq 0 < p_I^D(a_2, p_I) < p_E^S(p_I)$. This implies that Π_E is decreasing for all $p_I \geq 0$. Hence we can now state E's best response formally as

$$\hat{p}_I(a_2, p_I) = \begin{cases} 0 & for \quad 0 \leq p_I < \theta_0(a_2) \\ p_E^D & for \quad \theta_0(a_2) \leq p_I < \theta_1(a_2) \\ 0 & for \quad \theta_1(a_2) \leq p_I \end{cases} \tag{A.3.12}$$

where

$$\theta_0(a_2) = \frac{a_2 - 1 - c}{\sigma}.$$

iii) $\alpha_2^2 > a_2 \geq \alpha_2^3$: Because $a_2 \geq \alpha_2^3$, $\lambda_I(a_2) > \tau_I(a_2)$. This implies $p_E^M(a_2) \leq p_E \leq p_E^D(a_2, p_I)$ for $\tau_I(a_2) \leq p_I \leq \lambda_I(a_2)$. Notice that $\tau_I(\alpha_2^2) = 1 \Leftrightarrow p_E^M(\alpha_2^2) = 0$. Hence for $p_I < 1$, $p_E^S(p_I) < 0 < p_I^M(a_2) < p_I(a_2, p_I)$. However since $\Pi_E = \Pi_E^D$ for $p_E \geq 0$ and $p_I \leq 1$, the analysis does not change for $0 \leq p_I \leq 1$ as compared to the previous case. Recall that $\alpha_2^3 > \alpha_2^4$, which implies that $p_E^D(a_2, 0) < 0$. Therefore, for $0 \leq p_I \leq \theta_0(a_2)$, $\hat{p}_E(a_2, p_I) = 0$ and for $\theta_0(a_2) \leq p_I \leq 1$, $\hat{p}_E(a_2, p_I) = p_E^D(a_2, p_I)$.

Notice that for $p_I \in [1, \tau_I(a_2)]$, $0 \leq p_E^S(p_I) < p_I^M(a_2) < p_I(a_2, p_I)$ and therefore $\hat{p}_E(a_2, p_I) = p_E^D(a_2, p_I)$. However for $\tau_I(a_2) < p_I \leq \lambda_I(a_2)$, $p_E^M(a_2) < p_E^S(p_I) \leq p_E^D(a_2, p_I)$. This is to say that Π_E is now bi-modal, which implies that the optimal strategy for E is either $p_E^M(a_2)$ or $p_E^D(a_2, p_I)$. Recall that both are non-negative. Let us consider the function

$$\Gamma_1(p_I, a_2) = \Pi_E^M(p_I, p_E^M(a_2)) - \Pi_E^D(p_I, p_E^D(a_2, p_I)).$$

Notice that if $\Gamma_1(a_2, p_I) > 0$, $\hat{p}_E(a_2, p_I) = p_E^D(a_2, p_I)$. For $\Gamma_1(a_2, p_I) < 0$, $\hat{p}_E(a_2, p_I) = p_E^M(a_2)$. It is easy to see that $\Gamma_1(a_2, \tau_I(a_2)) < 0$ and $\Gamma_1(a_2, \lambda_I(a_2)) > 0$. Notice that for $p_I = \tau_I(a_2)$, $p_E^S(p_I) = p_E^M(a_2)$, hence $\frac{\partial \Pi_E}{p_E} > 0$ for $0 \leq p_E \leq p_E^S(p_I)$. However because $\lambda_I(a_2) \geq \tau_I(a_2)$, $p_E^D(a_2, \tau_I(a_2)) \geq p_E^S(\tau_I(a_2))$, hence Π_E is increasing in p_E up to $p_E^D(a_2, p_I)$. This implies that $\Gamma_1(a_2, \tau_I(a_2)) < 0$. At $p_I = \lambda_I(a_2)$, $p_E^S(p_I) = p_E^D(a_2, p_I)$, implying that for $\frac{\partial \Pi_E}{p_E} < 0$ for $p_E > p_E^S(p_I)$. Furthermore, because $\lambda_I(a_2) \leq \tau_I(a_2)$, $p_E^M(a_2) \leq p_E^S(\lambda_I(a_2))$. Hence Π_E is decreasing for $p_E^M(a_2) < p_E \leq p_E^S(p_I)$ and $\Gamma(a_2, \lambda_I(a_2)) > 0$. It is easy to see that $\Gamma_1(a_2, p_I)$ is concave and quadratic in p_I, because of the shapes of Π_E^M and Π_E^D. This implies that $\Gamma_1(a_2, p_I) = 0$ has two solutions in p_I at most. Finding the smaller of these roots $\theta_2(a_2)$ identifies intervals, $[\tau_I(a_2), \theta_2(a_2)]$, where $\hat{p}_E(a_2, p_I) = p_E^D(a_2, p_I)$ and $[\theta_2(a_2), \lambda_I(a_2)]$ where $\hat{p}_E(a_2, p_I) = p_E^M(a_2)$. It is straightforward to verify that

$$\theta_2 = \frac{c + a_2 - 1}{\sigma} - \frac{1 + \sigma + c\sigma^2 - c - a_2}{\sigma\sqrt{1-\sigma^2}}$$

Notice that $\theta_2(a_2) \in [\tau_I(a_2), \lambda_I(a_2)]$. The second root of $\Gamma_1(a_2, p_I) = 0$ is irrelevant, because since $\Gamma_1(a_2, \lambda_I(a_2)) > 0$, it is greater than $\lambda_I(a_2)$, where $p_E^M(a_2) < p_E^D(a_2, p_I) < p_E^S(a_2)$. Hence for $p_I \geq \lambda_I(a_2)$, $\hat{p}_E(a_2, p_I) = p_E^M(a_2)$. This case can be summed up by the following best response curve

$$\hat{p}_E = \begin{cases} 0 & for \quad 0 \leq p_I < \theta_0(a_2) \\ p_E^D & for \quad \theta_0(a_e) \leq p_I < \theta_2(a_2) \\ p_E^M & for \quad \theta_2(a_2) \leq p_I \end{cases} \tag{A.3.13}$$

iv) $\alpha_2^3 > a_2 \geq \alpha_2^4$: Notice that this implies $\theta_0(a_2) \geq 0$ since $a_2 \geq \alpha_2^4$. Additionally $\lambda_I(a_2) > 1$ and $\tau_I(a_2) > 1$, hence for $0 \leq p_I \leq \theta_0(a_2)$, $p_E^S(p_I) < p_E^D(a_2, p_I) \leq 0 < p_E^M(a_2)$. For $\theta_0(a_2) < p_I \leq 1$, $p_E^S(p_I) \leq 0 < p_E^D(a_2, p_I) < p_E^M(a_2)$. Hence $\hat{p}_E(a_2, p_I) = 0$ for $0 \leq p_I \leq \theta_0(a_2)$ and $\hat{p}_E(a_2, p_I) = p_E^D(a_2, p_I)$ for all $\theta_0(a_2) < p_I \leq 1$.

Furthermore, since $a_2 < \alpha_2^3$, $\lambda_I(a_2) < \tau_I(a_2)$. Therefore, for $1 < p_I \leq \lambda_I(a_2)$, $0 < p_E^S(p_I) \leq p_E^D(a_2, p_I) < p_E^M(a_2)$ and $\hat{p}_E(a_2, p_I) = p_E^D(a_2, p_I)$. For $\lambda_I(a_2) < p_I \leq \tau_I(a_2)$, $0 < p_E^D(a_2, p_I) < p_E^S(p_I) \leq p_E^M(a_2)$. Note that neither $p_E^M(a_2)$ nor $p_E^D(a_2, p_I)$ are in the feasible set of E. However that is to say that $\Pi_E = \Pi_E^M$ is increasing in p_E for $p_E \leq p_E^S(p_I)$ and $\Pi_E = \Pi_E^D$ is decreasing in p_E for $p_E > p_E^S(p_I)$. This in turn implies that Π_E is single-peaked at $p_E^S(p_I)$, which is then E's global profit maximum. Hence $\hat{p}_E(a_2, p_I) = p_E^S(p_I)$ for $p_I \in [\lambda_I(a_2), \tau_I(a_2)]$.

If $p_I > \tau_I(a_2)$, $0 < p_E^D(a_2, p_I) < p_E^S(p_I)$ and $p_E^M(a_2) < p_E^S(p_I)$, which implies a unique profit maximum at $p_E^M(a_2)$, i.e. $\hat{p}_E(a_2, p_I) = p_E^M(a_2)$. Summing up, E's best response for $\alpha_2^3 > a_2 \geq \alpha_2^4$ is given by

$$\hat{p}_E = \begin{cases} 0 & for \quad 0 \leq p_I < \theta_0(a_2) \\ p_E^D & for \quad \theta_0(a_2) \leq p_I < \lambda_I(a_2) \\ p_E^S & for \quad \lambda_I(a_2) \leq p_I < \tau_I(a_2) \\ p_E^M & for \quad \tau_I(a_2) \leq p_I \end{cases} \tag{A.3.14}$$

v) $\alpha_2^4 > a_2 \geq 0$: Notice that since $a_2 < \alpha_2^3$, $\lambda_I(a_2) < \tau_I(a_2)$, hence the analysis is similar to that of case iv). However $a_2 < \alpha_2^4$, which implies that $p_E^D(a_2, 0) > 0$, i.e. $\theta_0(a_2) < 0$. This implies that the best response of E for $\alpha_2^4 > a_2 \geq 0$ is given by

$$\hat{p}_E = \begin{cases} p_E^D & for \quad \theta_0(a_2) \leq p_I < \lambda_I(a_2) \\ p_E^S & for \quad \lambda_I(a_2) \leq p_I < \tau_I(a_2) \\ p_E^M & for \quad \tau_I(a_2) \leq p_I \end{cases} \tag{A.3.15}$$

The five different cases characterize E's best response to any $p_I \geq 0$ for different a_2. It has to be noted that neither best response depends on a_1. This actually implies that there 25 different combinations of I and E's best responses, with each exhibiting a possibly different equilibrium in prices. The next section analyzes these possibilities.

Pricing Equilibrium

This section derives the pricing equilibrium in the 2^{nd} stage of the game, given the findings of the previous subsection. For doing that, the following lemma turns out to be helpful

Lemma A.3.1. *Let $\hat{p}_I(a_1, p_E)$ and $\hat{p}_E(a_2, p_I)$ be the best response functions of I and E respectively as given in the previous sections. Then*

$$\hat{p}_I(\tilde{a}_1, p_E) \geq \hat{p}_I(a_1, p_E) \qquad \forall a_1 < \tilde{a}_1 \qquad \text{and} \qquad p_E \geq 0$$
$$\hat{p}_E(\tilde{a}_2, p_I) \geq \hat{p}_E(a_2, p_I) \qquad \forall a_2 < \tilde{a}_2 \qquad \text{and} \qquad p_I \geq 0$$

Proof. Let us start with firm I. Fix $\alpha_1^4 \leq \tilde{a}_1 \leq 0$. This immediately implies $\lambda_E(\tilde{a}_1) \leq \tau_E(\tilde{a}_1)$ and the best response function is given as in case v). Recall that for $0 \leq p_E \leq \lambda_E(\tilde{a}_1)$, $\hat{p}_I(\tilde{a}_1, p_E) = p_I^D(\tilde{a}_1, p_E)$, for $\lambda_E(\tilde{a}_1) < p_E \leq \tau_E(\tilde{a}_1)$, $\hat{p}_I(\tilde{a}_1, p_E) = p_I^S(p_E)$ and for $\tau(\tilde{a}_1) < p_E$, $\hat{p}_I(\tilde{a}_1, p_E) = p_I^M(a_1)$. Consider $a_1 < \tilde{a}_1$ in $[\alpha_1^4, 0]$. Note that $\frac{\partial \lambda_E}{\partial a_1} > 0$ and $\frac{\partial \tau_E}{\partial a_1} > 0$. Additionally increasing a_1 shifts up $p_I^D(a_1, p_E)$. Moreover by definition of $\lambda_E(a_1)$, $p_I^D(a_1, p_E) \geq p_I^S(p_I)$ for $p_E \leq \lambda_E(a_1)$. Since $p_I^S(p_E)$ is independent of a_1, this implies $\hat{p}_I(\tilde{a}_1, p_E) \geq \hat{p}_I(a_1, p_E)$ for all $a_1 < \tilde{a}_1$ in $[\alpha_1^4, 0]$ and $0 \leq p_E \leq \lambda(\tilde{a}_1)$. Notice that for $a_1 < \tilde{a}_1$ it is well possible that $\tau_E(a_1, p_E) < \lambda_E(\tilde{a}_1)$. However for $p_E \geq \tau_E(a_1)$, $p_I^M(a_1) \leq p_I^S(p_E)$, hence $\hat{p}_I(\tilde{a}_1, p_E) \geq \hat{p}_I(a_1, p_E)$ is indeed satisfied for $0 \leq p_E \leq \lambda_E(\tilde{a}_1)$.

Consider $\lambda_E(\tilde{a}_1) < p_E \leq \tau_E(\tilde{a}_1)$. For $a_1 < \tilde{a}_1$, $\lambda_E(a_1) < \lambda_E(\tilde{a}_1)$ as well as $\tau_E(a_1) < \tau_E(\tilde{a}_1)$. Notice that $p_I^M(a_1) \leq p_I^S(a_1)$ for $\tau_E(a_1) \leq p_E$ and $p_I^S(p_E)$ is independent of a_1, hence $\hat{p}_I(\tilde{a}_1, p_E) \geq \hat{p}_I(a_1, p_E)$ for all $a_1 < \tilde{a}_1$ in $[\alpha_1^4, 0]$ and $0 \leq p_E \leq \tau(\tilde{a}_1)$.

Finally we look at $\tau_E(\tilde{a}_1) < p_E$. For $a_1 < \tilde{a}_1$, $\tau_E(a_1) < \tau_E(\tilde{a}_1)$ and $p_I^M(a_1) < p_I^M(\tilde{a}_1)$. Hence $\hat{p}_I(\tilde{a}_1, p_E) \geq \hat{p}_I(a_1, p_E)$ for all $a_1 < \tilde{a}_1$ in $[\alpha_1^4, 0]$ and $p_E \geq 0$.

Suppose $\tilde{a}_1 \in [\alpha_1^3, \alpha_1^4]$. Notice that the best response curve as given by case iv) only differs from that in case v) for $0 \leq p_E \leq \eta_0(\tilde{a}_1)$, where $\hat{p}_I(\tilde{a}_1, p_E) = 0$. Since $\frac{\partial \eta_0}{\partial a_1} < 0$ and $p_I^D(\tilde{a}_1, p_E) \geq 0$ for $p_E \geq \eta_0(\tilde{a}_1)$, we can conclude that $\hat{p}_I(\tilde{a}_1, p_E) \geq \hat{p}_I(a_1, p_E)$ for all $a_1 < \tilde{a}_1$ in $[\alpha_1^3, 0]$ and $p_E \geq 0$.

Now fix $\tilde{a}_1 \in [\alpha_1^2, \alpha_1^3]$. $\hat{p}_I(\tilde{a}_1, p_E) = 0$ for $0 \leq p_E \leq \eta_0(\tilde{a}_1)$, $\hat{p}_I(\tilde{a}_1, p_E) = p_I^D(\tilde{a}_1, p_E)$ for $\eta_0(\tilde{a}_1) < p_E \leq \eta_2(\tilde{a}_1)$ and $\hat{p}_I(\tilde{a}_1, p_E) = p_I^M(\tilde{a}_1)$ for $\eta_2(\tilde{a}_1) < p_E$. Notice that $\frac{\partial \eta_2}{\partial a_1} > 0$. Consider $a_1 < \tilde{a}_1$ in $[\alpha_1^2, \alpha_1^3]$. Observe that $\frac{\partial \eta_0}{\partial a_1} < 0$ hence $\hat{p}_I(\tilde{a}_1, p_E) = \hat{p}_I(a_1, p_E)$ for all $a_1 < \tilde{a}_1$ in $[\alpha_1^3, 0]$ and $0 \leq p_E \leq \eta_0(\tilde{a}_1)$.

Consider $\eta_0(\tilde{a}_1) < p_E \leq \eta_2(\tilde{a}_1)$. Because $\frac{\partial \eta_2}{\partial a_1} > 0$, for $a_1 < \tilde{a}_1$ in $[\alpha_1^2, \alpha_1^3]$, $\{\eta_0(a_1), \eta_2(a_1)\} \in [\eta_0(\tilde{a}_1), \eta_2(\tilde{a}_1)]$. Notice that $p_I^D(\tilde{a}_1, p_E) > p_I^D(a_1, p_E)$ for all $a_1 < \tilde{a}_1$, and $p_E \geq 0$. Moreover, $p_I^M(a_1) \leq p_I^D(a_1, \eta_2(a_1))$ for $a_1 \in [\alpha_1^2, \alpha_1^3]$, hence $\hat{p}_I(\tilde{a}_1, p_E) > \hat{p}_I(a_1, p_E)$ for $\eta_0(\tilde{a}_1) < p_E \leq \eta_2(\tilde{a}_1)$.

It easy to see that $p_I^M(a_1) < p_I^M(\tilde{a}_1)$ for $a_1 < \tilde{a}_1$, hence $\hat{p}_I(\tilde{a}_1, p_E) > \hat{p}_I(a_1, p_E)$ for $p_E \geq 0$ for all $a_1 < \tilde{a}_1$ in $[\alpha_1^2, \alpha_1^3]$. Notice that $p_I^M(\alpha_1^3) = p_I^D(\alpha_1^3, \eta_2(\alpha_1^3)) = p_I^D(\alpha_1^3, \lambda_E(\alpha_1^3))$. Therefore $\hat{p}_I(\tilde{a}_1, p_E) \geq \hat{p}_I(a_1, p_E)$ for $p_E \geq 0$ for all $a_1 < \tilde{a}_1$ in $[\alpha_1^2, 0]$.

Set $\tilde{a}_1 \in [\alpha_1^1, \alpha_1^2]$. $\hat{p}_I(\tilde{a}_1, p_E) = 0$ for $0 \leq p_E \leq \eta_0(\tilde{a}_1)$ and $\eta_1(\tilde{a}_1) \leq p_E$, $\hat{p}_I(\tilde{a}_1, p_E) = p_I^D(\tilde{a}_1, p_E)$ for $\eta_0(\tilde{a}_1) < p_E < \eta_1(\tilde{a}_1)$. Consider $a_1 < \tilde{a}_1$. Notice that $\{\eta_0(a_1), \eta_1(a_1)\} \in [\eta_0(\tilde{a}_1), \eta_1(\tilde{a}_1)]$, because $\frac{\partial \eta_0}{\partial a_1} < 0$ and $\frac{\partial \eta_1}{\partial a_1} > 0$. Hence $\hat{p}_I(\tilde{a}_1, p_E) = \hat{p}_I(a_1, p_E)$ for $0 \leq p_E \leq \eta_0(\tilde{a}_1)$ and $\eta_1(\tilde{a}_1) \leq p_E$. Moreover since $p_I^D(\tilde{a}_1, p_E) > p_I^D(a_1, p_E)$ for all $a_1 < \tilde{a}_1$ and $p_E \geq 0$, $\hat{p}_(\tilde{a}_1, p_E) \geq \hat{p}_I(a_1, p_E)$ for $p_E \geq 0$. Notice that $\eta_1(\alpha_1^2) = \eta_2(\alpha_1^2)$ and $p_I^M(\alpha_1^2 = 0)$, therefore $\hat{p}_(\tilde{a}_1, p_E) \geq \hat{p}_I(a_1, p_E)$ for $p_E \geq 0$ and $a_1 < \tilde{a}_1$ in $[\alpha_1^1, 0]$.

Now for all $a_1 < \alpha_1^1$, $\hat{p}_I(a_1, p_E) = 0$. Since $\hat{p}_I(a_1, p_E) \geq 0$ for all $a_1 \in [\alpha_1^1, 0]$ we can conclude that $\hat{p}_I(a_1, p_E) \geq \hat{p}_I(\tilde{a}_1, p_E)$ for all $a_1 < \tilde{a}_1$ in $[-\infty, 0]$ and $\hat{p}_I(0, p_E)$ is indeed an upper envelope for all $a_1 < 0$ and $p_E \geq 0$.

Let us now turn to E's best response functions. We start with fixing $\tilde{a}_2 \in [0, \alpha_2^4]$. This immediately implies that E's best response is as given in case v), i.e. $\hat{p}_E(\tilde{a}_2, p_I) = p_E^D(\tilde{a}_2, p_I)$ for $0 \leq p_I \leq \lambda_I(\tilde{a}_2)$, $\hat{p}_E(\tilde{a}_2, p_I) = p_E^S(p_I)$ for $\lambda_I(\tilde{a}_2) < p_I \leq \tau_I(\tilde{a}_2)$ and $\hat{p}_E(\tilde{a}_2, p_I) = p_E^M(\tilde{a}_2)$ for $p_I > \tau_I(\tilde{a}_2)$. Consider $a_2 > \tilde{a}_2$ in $[0, \alpha_2^4]$. Recall that $\frac{\partial \lambda_I}{\partial a_2} < 0$ as well as $\frac{\partial \tau_I}{\partial a_2} < 0$, hence $\lambda_I(a_2) < \lambda_I(\tilde{a}_2)$. Since $p_E^D(\tilde{a}_2, p_I) > p_E^D(a_2, p_I)$ for $a_2 > \tilde{a}_2$ and by definition of $\lambda_I(a_2)$, $p_E^D(\tilde{a}_2, p_I) \geq p_E^S(p_I)$ for $p_I \leq \lambda_I(\tilde{a}_2)$, it follows that $\hat{p}_E(\tilde{a}_2, p_I) \geq \hat{p}_E(a_2, p_I)$ for $0 \leq p_I \leq \lambda_I(\tilde{a}_2)$. Notice that $\tau_I(a_2) \leq \lambda_I(\tilde{a}_2)$ is also possible. However since $p_E^M(a_2) \leq p_E^S(p_I)$ for $\tau_I(a_2) \leq p_I$, $\hat{p}_E(\tilde{a}_2, p_I) \geq \hat{p}_E(a_2, p_I)$ is still satisfied for $0 \leq p_I \leq \lambda_I(\tilde{a}_2)$.

Consider $\lambda_I(\tilde{a}_2) < p_I \leq \tau_I(\tilde{a}_2)$. Because $p_E^S(p_I)$ is independent of a_2 and $p_E^M(a_2) \leq p_E^S(p_I)$ for $\tau_I(a_2) \leq p_I$, we have $\hat{p}_E(\tilde{a}_2, p_I) \geq \hat{p}_E(a_2, p_I)$ for any $a_2 > \tilde{a}_2$ in $[0, \alpha_2^4]$.

For $\tau_I(\tilde{a}_2) < p_I$, $\hat{p}_E(\tilde{a}_2, p_I) = p_E^M(\tilde{a}_2)$. Because $\frac{\partial p_E^M}{\partial a_2} < 0$, $\hat{p}_E(\tilde{a}_2, p_I) > \hat{p}_E(a_2, p_I)$ for $a_2 > \tilde{a}_2$ in $[0, \alpha_2^4]$ and $p_I \geq 0$.

Notice that case v) and case iv) best responses are similar except for $0 \leq p_I \leq \theta_0(a_1)$. Hence fixing $\tilde{a}_2 \in [\alpha_2^3, \alpha_2^4]$ yields exactly the same results as discussed before. Since $\frac{\partial \theta_0}{\partial a_2} > 0$, $\hat{p}_E(\tilde{a}_2, p_I) = \hat{p}_E(a_2, p_I) = 0$ for all $a_2 > \tilde{a}_2$ and $0 \leq p_I \leq \theta_0(a_2)$. However since $p_E^D(\tilde{a}_2, p_I) \geq 0$ for $\theta_0(\tilde{a}_2) < p_I$, and $p_E^D(\tilde{a}_2, p_I) > p_E^D(a_2, p_I)$ for $a_2 > \tilde{a}_2$, we can conclude that $\hat{p}_E(\tilde{a}_2, p_I) \geq \hat{p}_E(a_2, p_I)$ for $a_2 > \tilde{a}_2$ in $[\alpha_2^4, \alpha_2^3]$ and $p_I \geq 0$. Moreover because $\theta_0(\alpha_2^4) = 0$, this is true for all $a_2 > \tilde{a}_2$ in $[0, \alpha_2^3]$.

If $\tilde{a}_2 \in [\alpha_2^3, \alpha_2^2]$, E's best response is given by $\hat{p}_(\tilde{a}_2, p_I) = 0$ for $0 \leq p_I \leq \theta_0(\tilde{a}_2)$, $\hat{p}_E(\tilde{a}_2, p_I) = p_E^D(\tilde{a}_2, p_I)$ for $\theta_0(\tilde{a}_2) < p_I \leq \theta_2(\tilde{a}_2)$ and $\hat{p}_E(\tilde{a}_2, p_I) = p_E^M(\tilde{a}_2)$ for $\theta_2(\tilde{a}_2) < p_I$. Since $\frac{\partial \theta_0}{\partial a_2} > 0$ and $\frac{\partial \theta_2}{\partial a_2} < 0$, $\{\theta_0(a_2), \theta_2(a_2)\} \in [\theta_0(\tilde{a}_2), \theta_2(\tilde{a}_2)]$ for $a_2 > \tilde{a}_2$ in $[\alpha_2^3, \alpha_2^2]$. For $0 \leq p_I \leq \theta_0(\tilde{a}_2)$, $\hat{p}_E(\tilde{a}_2, p_I) = \hat{p}_E(a_2, p_I) = 0$.

Notice that $p_E^D(\tilde{a}_2, p_I) > 0$ for $\theta_0(\tilde{a}_2) < p_I \leq \theta_2(\tilde{a}_2)$ and $p_E^D(\tilde{a}_2, p_I) > p_E^D(a_2, p_I)$ for $a_2 > \tilde{a}_2$. Moreover $p_E^D(a_2, \theta_2(a_2)) \geq p_I^M(a_2)$ for $a_2 \in [\alpha_2^3, \alpha_2^2]$ and $p_E^M(\tilde{a}_2) > p_E^M(a_2)$ for

$a_2 > \tilde{a}_2$. Hence $\hat{p}_E(\tilde{a}_2, p_I) \geq \hat{p}_E(a_2, p_I)$ for $a_2 > \tilde{a}_2$ in $[\alpha_2^3, \alpha_2^2]$ and $p_I \geq 0$.

Furthermore $p_E^M(\alpha_2^3) = p_E^D(\alpha_2^3, \theta_2(\alpha_2^3)) = p_E^D(\alpha_2^3, \lambda_I(\alpha_2^3))$ and $\lambda_I(\alpha_2^3) = \tau_I(\alpha_2^3)$ imply that $\hat{p}_E(\tilde{a}_2, p_I) \geq \hat{p}_E(a_2, p_I)$ for $a_2 > \tilde{a}_2$ in $[0, \alpha_2^2]$ and $p_I \geq 0$.

Set $\tilde{a}_2 \in [\alpha_2^2, \alpha_2^1]$. $\hat{p}_E(\tilde{a}_2, p_I) = 0$ for $0 \leq p_I \leq \theta_0(\tilde{a}_2)$ and $\theta_1(\tilde{a}_2) \leq p_I$. Since $\frac{\partial \theta_0}{\partial a_2} > 0$ and $\frac{\partial \theta_1}{\partial a_2} < 0$, $\{\theta_0(a_2), \theta_1(a_2)\} \in [\theta_0(\tilde{a}_2), \theta_1(\tilde{a}_2)]$ for $a_2 > \tilde{a}_2$ in $[\alpha_2^2, \alpha_2^1]$, hence $\hat{p}_E(\tilde{a}_2, p_I) = \hat{p}_E(a_2, p_I) = 0$ for $0 \leq p_I \leq \theta_0(a_2)$ and $\theta_1(\tilde{a}_2) \leq p_I$. Since $p_E^D(\tilde{a}_2, p_I) > 0$ for $\theta_0(\tilde{a}_2) < p_I \leq \theta_2(\tilde{a}_2)$ and $p_E^D(\tilde{a}_2, p_I) > p_E^D(a_2, p_I)$ for $a_2 > \tilde{a}_2$, $\hat{p}_E(\tilde{a}_2, p_I) \geq \hat{p}_E(a_2, p_I)$ for $a_2 > \tilde{a}_2$ in $[\alpha_2^2, \alpha_2^1]$ and $p_I \geq 0$. Moreover because $p_E^D(\alpha_2^2, \theta(\alpha_2^2) = p_E^D(\alpha_2^2, \theta_2(\alpha_2^2)$ and $p_E^M(\alpha_2^2) = 0$, this holds for $a_2 > \tilde{a}_2$ in $[0, \alpha_1^1]$.

Now for $a_2 > \alpha_2^1$, $\hat{p}_E(a_2, p_I) = 0$. Since $\hat{p}_E(a_2, p_I) \geq 0$ for all $a_2 \in [0, \alpha_2^1]$ we can conclude that $\hat{p}_E(\tilde{a}_2, p_I) \geq \hat{p}_E(a_2, p_I)$ for all $a_2 < \tilde{a}_2$ in $[0, \infty]$ and $\hat{p}_E(0, p_I)$ is indeed an upper envelope for all $a_2 > 0$ and $p_I \geq 0$. ■

Identifying upper envelopes for I and E also means identifying a set of price-tuples (p_I, p_E), that are candidate pricing equilibria. This set is defined as the intersection of

$$\hat{p}_I(0, p_E) \leq \hat{p}_I(a_1, p_E) \qquad \forall a_1 < 0$$

$$\hat{p}_E(0, p_I) \leq \hat{p}_E(a_2, p_I) \qquad \forall a_2 > 0$$

Identifying this set is equivalent to finding an intersection points of $\hat{p}_I(0, p_E)$ and $\hat{p}_E(0, p_I)$. In order to find these points we can proceed piecewise, this means for every interval of the two best response functions. Lemma A.3.2 and ??? summarize the results of this exercise.

Lemma A.3.2. *Consider $\hat{p}_I(0, p_E)$ and $\hat{p}_E(0, p_I)$ for $p_I \leq p_I^D(0, \lambda_E(0))$ and $p_E \leq p_E^D(0, \lambda_I(0))$. The unique intersection point of $\hat{p}_I(0, p_E)$ and $\hat{p}_E(0, p_I)$ is given by*

$$\begin{aligned} p_I^*(0,0) &= \frac{1}{2-\sigma} + \frac{c(4-\sigma)}{4-\sigma^2} \\ p_E^*(0,0) &= \frac{1}{2-\sigma} + \frac{c(\sigma(2-\sigma)+2)}{4-\sigma^2} \end{aligned}$$

Proof. Consider the functions $p_I^D(a_1, p_E)$ and $p_E^D(a_2, p_I)$. Simple algebra shows that these intersect at

$$p_I^*(a_1, a_2) = \frac{1}{2-\sigma} + \frac{c(4-\sigma)}{4-\sigma^2} + \frac{2a_1 - \sigma a_2}{4-\sigma^2} \tag{A.3.16}$$

$$p_E^*(a_1, a_2) = \frac{1}{2-\sigma} + \frac{c(\sigma(2-\sigma)+2)}{4-\sigma^2} + \frac{\sigma a_1 - 2a_2}{4-\sigma^2} \tag{A.3.17}$$

It is easy to verify, $p_I^*(a_1, a_2) \geq 0$ and $p_E^*(a_, a_2) \geq 0$ if

$$a_1 \geq \frac{\sigma}{2}\Big(c + a_2 - 1\Big) - 1 - 2c \tag{A.3.18}$$

$$a_2 \leq \frac{\sigma}{2}\Big(1 + a_1 - c\sigma\Big) + 1 + c\sigma + c \tag{A.3.19}$$

These inequalities are certainly satisfied for $a_1 = 0$ and $a_2 = 0$. Additionally, for $a_1 = a_2 = 0$,

$$p_I^*(0,0) < p_I^D(0, \lambda_E(0)) \quad \Leftrightarrow \quad \frac{\sigma(2+\sigma-2c(1-\sigma))}{(4-\sigma^2)(2-\sigma^2)} > 0$$

$$p_E^*(0,0) < p_E^D(0, \lambda_I(0)) \quad \Leftrightarrow \quad \frac{\sigma\Big(2+\sigma-c\big(4+\sigma(1-\sigma)\big)\big(1-\sigma\big)\Big)}{(4-\sigma^2)(2-\sigma^2)} > 0$$

which is satisfied for $\sigma \in [\frac{1}{2}, 1]$ and $c \in [0, \frac{1}{2}]$. It is easy to see that

$$\lambda_I(0) > p_I^D(0, \lambda_E(0))$$
$$\lambda_E(0) > p_E^D(0, \lambda_I(0)),$$

which is to say that for $p_I \leq p_I^D(0, \lambda_E(0))$ and $p_E \leq p_E^D(0, \lambda_I(0))$, it is only $p_I^D(0, p_E)$ and $p_E^D(0, p_I)$ that we care about in the best response function. Hence because of the linearity of $p_I^D(0, p_E)$ and $p_E^D(0, p_I)$, we can conclude that lemma A.3.2 is indeed true. ∎

Notice that lemma A.3.2 implies that a set of equilibrium candidates is given by

$$0 \leq p_I \leq p_I^D(0, p_E) \tag{A.3.20}$$

$$0 \leq p_E \leq p_E^D(0, p_I). \tag{A.3.21}$$

To show that this is the only set of candidate equilibria, consider lemma A.3.3.

Lemma A.3.3. *$\hat{p}_I(0, p_E)$ and $\hat{p}_E(0, p_I)$ cross only once for $p_I \geq 0$ and $p_E \geq 0$.*

Proof. Since Lemma A.3.2 provides an intersection point for $p_I \leq p_I^D(0, \lambda_E(0))$ and $p_E \leq p_E^D(0, \lambda_I(0))$, we have to show that $\hat{p}_I(0, p_E)$ and $\hat{p}_E(0, p_I)$ cannot cross in the rest of the positive quadrant. This can be done by observing the different parts of E and I's best response curves and their possible combinations.

It is obvious that $p_I^D(a_1, 0) < 1$ for all $a_1 \leq 0$ as well as $p_E^D(a_2, 0) < 1$ for all $a_2 \geq 0$. Furthermore we have $\frac{\partial p_I^D}{\partial p_E} = \sigma/2 < \left[\frac{\partial p_E^S}{\partial p_I}\right]^{-1} = \sigma$ and $\frac{\partial p_I^S}{\partial p_E} = 1/\sigma < \left[\frac{\partial p_E^D}{\partial p_I}\right]^{-1} = 2/\sigma$. These conditions ensure that $p_I^D(a_1, p_E)$ and $p_E^S(p_I)$ as well as $p_E^D(a_2, p_I)$ and $p_I^S(p_E)$ can never cross in the northwest quadrant, that is with $p_I \geq 0$ and $p_E \geq 0$.

Additionally we can rule out the intersection of $p_I^S(p_E)$ and $p_E^S(p_I)$. This occurs at $p_I = p_E = \frac{1}{1-\sigma}$. However since $\tau_E(0) < \frac{1}{1-\sigma}$ and $\tau_I(0) < \frac{1}{1-\sigma}$, this intersection point is never in the feasible set for any $a_1 \leq 0$ and $a_2 \geq 0$.

This also implies that the best response function do not cross for $p_E \geq \tau_E(a_1)$ and $p_I \geq \tau_I(a_2)$ because of their shape. Hence Lemma A.3.2 yields the unique intersection point of $\hat{p}_I(0, p_E)$ and $\hat{p}_E(0, p_I)$. ■

We can therefore conclude that (A.3.20) and (A.3.21) describe the only set of candidate equilibria of the pricing game. It also implies that $p_I^S(p_E)$ and $p_E^S(p_I)$ can never part of an equilibrium, as well as $p_I^M(a_1)$ and $p_E^M(a_2)$ for $a_1 \in [\alpha_1^3, 0]$ and $a_2 \in [0, \alpha_2^3]$.

Proposition A.3.1. *Suppose $\alpha_1^4 \leq a_1 \leq 0$ and $0 \leq a_2 \leq \alpha_2^4$. Best response functions of I and E are given by cases v). Then the equilibrium prices are given by*

$$\begin{aligned} p_I^*(a_1, a_2) &= \frac{1}{2-\sigma} + \frac{c(4-\sigma)}{4-\sigma^2} + \frac{2a_1 - \sigma a_2}{4-\sigma^2} \\ p_E^*(a_1, a_2) &= \frac{1}{2-\sigma} + \frac{c(\sigma(2-\sigma)+2)}{4-\sigma^2} + \frac{\sigma a_1 - 2a_2}{4-\sigma^2} \end{aligned}$$

Proof. Notice that Lemma A.3.3 implies that$p_I^M(a_1)$, $p_E^M(a_2)$ can not be part of the equilibrium for $a_1 \in [\alpha_1^3, 0]$ and $a_2 \in [0, \alpha_2^3]$ as well as $p_I^S(p_E)$ and $p_E^S(p_I)$. Hence we only have to look at $p_I^D(a_1, p_E)$ and $p_E^D(a_2, p_I)$. These have a unique intersection point at $p_I^*(a_1, a_2)$ and $p_E^*(a_1, a_2)$. ■

Proposition A.3.2. *Suppose $a_1 \leq \alpha_1^4$ and $\alpha_2^4 \leq a_2$. Best response functions of I and E are given by cases i)-iv). Then the equilibrium prices are given by*

$$p_I^* = 0 \quad \text{and} \quad p_E^* = 0$$

Proof. By lemma A.3.2 we know that $\hat{p}_I(\alpha_1^4, p_E) \geq \hat{p}_I(a_1, p_E) \,\forall\, a_1 \leq \alpha_1^4$ and $\hat{p}_E(\alpha_2^4, p_I) \geq \hat{p}_E(a_2, p_I) \;\forall\; a_2 \geq \alpha_2^4$. It is easy to verify that $p_I^*(\alpha_1^4, \alpha_2^4) = p_E^*(\alpha_1^4, \alpha_2^4) = 0$. Since we know that $p_I^*(a_1, a_2)$ and $p_E^*(a_1, a_2)$ is the unique intersection point of $p_I^D(a_1, p_E)$ and $p_E^D(a_2, p_I)$, $p_I^S(p_E)$ and $p_E^S(p_I)$ as well as $p_I^M(a_1)$, $p_E^M(a_2)$ can not be part of the equilibrium for $a_1 \in [\alpha_1^3, 0]$ and $a_2 \in [0, \alpha_2^3]$. Hence $\hat{p}_I(\alpha_1^4, p_E)$ and $\hat{p}_E(\alpha_2^4, p_I)$ cross only at at (0,0). This also implies that the only possible equilibrium is $p_I^* = p_E^* = 0$. Notice that for every $a_1 \leq \alpha_1^4$, (0,0) is part of the best response for I. The same is true for $a_2 \geq \alpha_2^4$ and E's best response, which proves the proposition. ■

Proposition A.3.3. *Suppose $0 \geq a_1 \geq \alpha_1^4$ and $a_2 \geq 1 + c(1+\sigma) + \frac{\sigma}{2}\big(1 - c\sigma + a_1\big)$. Then the equilibrium prices are given by*

$$p_I^* = p_I^D(a_1, 0) \quad \text{and} \quad p_E^* = 0$$

Proof. Notice that the restrictions in proposition A.3.3 imply $a_2 > \alpha_2^4$, which is to say that $\theta_0(a_2) \geq 0$. For $a_1 \geq \alpha_1^4$, $p_I^D(a_1, 0) \geq 0$. Hence there are two possible situations, either $p_I^D(a_1, 0) \leq \theta_0(a_2)$ or $p_I^D(a_1, 0) \geq \theta_0(a_2)$. It is easy to see that

$$p_I^D(a_1, 0) \leq \theta_0(a_2) \qquad \Leftrightarrow \qquad a_2 \geq 1 + c(1+\sigma) + \frac{\sigma}{2}\big(1 - c\sigma + a_1\big)$$

Moreover, by lemma A.3.2 and A.3.3 we know that $p_I^D(a_1, p_E)$ is the only branch, where an equilibrium could occur. Notice that

$$\left[\frac{\partial p_E^D}{\partial p_I}\right]^{-1} > \left[\frac{\partial p_E^S}{\partial p_I}\right]^{-1} > \frac{\partial p_I^D}{\partial p_E}.$$

Together with the restrictions on a_1 and a_2 this implies that $p_I^D(a_1, p_E) < \hat{p}_E^{-1}(a_1, p_E)$ for $0 < p_E \leq \lambda_E(a_1)$. At $p_E = 0$, $p_I^D(a_1, 0) > 0$. Furthermore, $\hat{p}_E(a_2, p_I) = 0$ for $0 \leq p_I \leq \theta_0(a_2)$. Hence the unique equilibrium is given by

$$p_I^* = p_I^D(a_1, 0) \qquad \text{and} \qquad p_E^* = 0$$

■

Proposition A.3.4. *Suppose* $0 \geq a_1 \geq \alpha_1^4$ *and* $\alpha_2^4 \leq a_2 < 1 + c(1+\sigma) + \frac{\sigma}{2}\big(1 - c\sigma + a_1\big)$. *Then the equilibrium prices are given by*

$$p_I^* = p_I^*(a_1, a_2) \quad \text{and} \quad p_E^* = p_E^*(a_1, a_2)$$

Proof. Notice that the parameter restrictions of proposition A.3.4 satisfy (A.3.18) and (A.3.19), hence $p_I^*(a_1, a_2) > 0$ and $p_E^*(a_1, a_2) > 0$. Hence a potential equilibrium is $p_I^* = p_I^*(a_1, a_2)$ and $p_E^* = p_E^*(a_1, a_2)$. However, it remains to be verified, whether this is always in I and E's feasible set, i.e. element of either best response curve.

It is easy to verify that $0 \leq p_E^*(a_1, a_2) \leq \lambda_E(a_1)$ for all $a_1 \leq 0$ and $0 \leq a_2$. By definition, $p_I^*(a_1, a_2) > p_I^D(a_1, 0) > \theta_0(a_2)$. Furthermore, $\theta_1(a_2) \geq 1$ as well as $\theta_2(a_2) > 1$. However notice that

$$p_I^*(a_1, a_2) > 1 \qquad \Leftrightarrow \qquad a_2 < \frac{-2 + \sigma(1+\sigma) + c(4-\sigma) + 2a_1}{\sigma} = \underline{a}_2(a_1)$$

Moreover it is easy to see that $p_I^*(a_1, a_2)$ is increasing in a_1. Hence if $\underline{a}_2(a_1) > a_2$ is satisfied for $a_1 = 0$, we know that it is also satisfied for all $a_1 < 0$. Simple algebra shows that $\underline{a}_2(0) < \alpha_2^4$. We can therefore conclude that for $\alpha_2^4 \leq a_2 < 1 + c(1+\sigma) + \frac{\sigma}{2}\big(1 - c\sigma + a_1\big)$, $p_I^*(a_1, a_2)$ is indeed an equilibrium.

Notice that $p_I^* = p_I^*(a_1, a_2)$ and $p_E^* p_E^*(a_1, a_2)$ is a unique equilibrium. $p_I^D(a_1, p_E)$ is the only relevant part of I's best response curve. Furthermore it is increasing in p_E and $p_E^D(a_2, \theta_1(a_2)) > 0$ as well as $p_E^D(a_2, \theta_1(a_2)) > p_E^M(a_2)$, which rules out any other possible intersection points of the best response functions. ■

Proposition A.3.5. *Suppose* $0 \leq a_2 \leq \alpha_2^4$ *and* $a_1 \leq -1 - 2c - \frac{\sigma}{2}\big(1 - c + a_1\big)$. *Then the equilibrium prices are given by*

$$p_I^* = 0 \quad \text{and} \quad p_E^* = p_E^D(a_2, 0)$$

Proof. It is easy to see that $0 \leq a_2 \leq \alpha_2^4$ and $a_1 \leq -1 - 2c - \frac{\sigma}{2}\big(1 - c + a_1\big)$ imply $a_1 < \alpha_1^4$. Hence $\eta_0(a_1) > 0$, moreover, $\eta_0(a_1) \geq p_E^D(a_2, 0)$. Notice that because of lemmas A.3.2 and A.3.3, only $p_E^D(a_2, p_I)$ is relevant for finding an equilibrium. However since

$$\left[\frac{\partial p_I^D}{\partial p_E}\right]^{-1} > \left[\frac{\partial p_I^S}{\partial p_E}\right]^{-1} > \frac{\partial p_E^D}{\partial p_I}.$$

Together with the restrictions on a_1 and a_2 this implies that $p_E^D(a_2, p_I) < \hat{p}_I^{-1}(a_2, p_E)$ for $0 < p_I \leq \lambda_I(a_2)$. However notice that for $p_E \in [0, \eta_0(a_1)]$, $\hat{p}_I(a_1, p_E) = 0$. Since $p_E^D(a_2, p_I) \in [0, \eta_0(a_1)]$, we can conclude that the unique equilibrium is given by

$$p_I^* = 0 \text{and} \qquad p_E^* = p_E^D(a_2, 0) \qquad \blacksquare$$

Proposition A.3.6. *Suppose* $0 \leq a_2 \leq \alpha_2^4$ *and* $\alpha_1^4 \geq a_1 > -1 - 2c - \frac{\sigma}{2}\big(1 - c + a_1\big)$. *Then the equilibrium prices are given by*

$$p_I^* = p_I^*(a_1, a_2) \quad \text{and} \quad p_E^* = p_E^*(a_1, a_2)$$

Proof. Since $0 \leq a_2 \leq \alpha_2^4$ and $\alpha_1^4 \geq a_1 > -1 - 2c - \frac{\sigma}{2}\big(1 - c + a_1\big)$ satisfy (A.3.18) and (A.3.19), a possible candidate equilibrium is $p_I^* = p_I^*(a_1, a_2)$ and $p_E^* = p_E^*(a_1, a_2)$. Notice that if this point is feasible, i.e. on both best response functions of I and E, it is also unique. $p_I^S(p_E)$ and $p_E^S(p_I)$ cannot be part of an equilibrium . Furthermore, $p_I^D(a_1, \eta_1(a_1) \geq 0$ for $\alpha_1^1 \geq a_1 \geq \alpha_1^2$ and $p_I^D(a_1, \eta_2(a_1)) \geq p_I^M(a_1)$ for $\alpha_1^2 \geq a_1 \geq \alpha_1^3$. Because $p_E^D(a_2, p_I)$ is increasing in p_I, $p_I^* = p_I^*(a_1, a_2)$ and $p_E^* = p_E^*(a_1, a_2)$ is the only point of intersection with $\hat{p}_I(a_1, p_E)$, if it is feasible.

By definition, $\eta_0(a_1) < p_E^D(a_2, 0)$. Because $\frac{\partial p_I^D}{\partial p_E} < \left[\frac{\partial p_E^D}{\partial p_I}\right]^{-1}$, $p_E^D(a_2, 0) < p_E^*(a_1, a_2)$. Moreover notice that $\eta_1(a_1) \geq 1$ as well as $\eta_2(a_1)$. But

$$p_E^*(a_1, a_2) > 1 \qquad \Leftrightarrow \qquad a_1 > \frac{2 - (\sigma + 1)(2c + \sigma) + 2a_2 + c\sigma^2}{\sigma} = \underline{a}_1(a_2)$$

It is easy to verify that $\frac{p_E^*}{\partial a_2} < 0$, hence it is sufficient to check for $a_2 = 0$. Simple algebra shows that

$$\underline{a}_1(0) = \frac{2 - (\sigma + 1)(2c + \sigma) + c\sigma^2}{\sigma} > \alpha_1^4$$

Hence, $p_E^*(a_1, a_2) > 1$ is only satisfied for $a_1 > \underline{a}_1(a_2) > \alpha_1^4$. Hence $p_E^*(a_1, a_2)$ is always feasible. It is also easy to see that $0 \leq p_I^*(a_1, a_2) \leq \lambda_I(a_2)$ for all $a_2 \geq 0$ and $a_1 \leq 0$. Therefore, $p_I^* = p_I^*(a_1, a_2)$ and $p_E^* = p_E^*(a_1, a_2)$ is the unique equilibrium. $\blacksquare$

1st Stage

After having identified all possible equilibria in the pricing stage, given a_1 and a_2, we can now turn to the first stage and look at the optimal choice of a_1 and a_2. Notice that proposition A.3.1 to A.3.6 divide (a_1, a_2)-space into four mutually exclusive sets. The following lemma identifies the subset that turns out relevant for identifying the optimal choice of a_1 and a_2.

Lemma A.3.4. *The optimal* $\{a_1^*, a_2^*\} \in \mathbb{K}_1$ *with*

$$\mathbb{K}_1 = \left(\{a_1, a_2\} | a_2 < 1 + c(1+\sigma) + \frac{\sigma}{2}\big(1 - c\sigma + a_1\big) \quad \wedge \quad a_1 > -1 - 2c - \frac{\sigma}{2}\big(1 - c + a_1\big)\right)$$

Proof. Depending on a_1 and a_2, four different equilibria in the pricing game are possible. This allows us to subdivide (a_1, a_2)-space into four mutually exclusive sets. The following relationships hold.

$$\begin{array}{llll}
\{a_1, a_2\} \in \mathbb{K}_1 & \Leftrightarrow & p_I^* = p_I^*(a_1, a_2) & p_E^* = p_E^*(a_1, a_2) \\
\{a_1, a_2\} \in \mathbb{K}_2 & \Leftrightarrow & p_I^* = p_I^D(a_1, 0) & p_E^* = 0 \\
\{a_1, a_2\} \in \mathbb{K}_3 & \Leftrightarrow & p_I^* = 0 & p_E^* = 0 \\
\{a_1, a_2\} \in \mathbb{K}_4 & \Leftrightarrow & p_I^* = 0 & p_E^* = p_E^D(a_2, 0)
\end{array}$$

with

$$\begin{aligned}
\mathbb{K}_1 &= \left(\{a_1, a_2\} | a_2 < 1 + c(1+\sigma) + \frac{\sigma}{2}\big(1 - c\sigma + a_1\big) \wedge a_1 > -1 - 2c - \frac{\sigma}{2}\big(1 - c + a_1\big)\right) \\
\mathbb{K}_2 &= \left(\{a_1, a_2\} | a_2 < 1 + c(1+\sigma) + \frac{\sigma}{2}\big(1 - c\sigma + a_1\big) \wedge a_1 > \alpha_1^4\right) \\
\mathbb{K}_3 &= \left(\{a_1, a_2\} | a_2 \geq \alpha_2^4 \wedge a_1 \leq \alpha_1^4\right) \\
\mathbb{K}_4 &= \left(\{a_1, a_2\} | a_2 < \alpha_2^4 \wedge a_1 > -1 - 2c - \frac{\sigma}{2}\big(1 - c + a_1\big)\right)
\end{aligned}$$

The allocation of pricing equilibria to different subsets in (a_1, a_2)-space has implications on the profit functions of I and E. The choice variables of the firms are a_1 and a_2, however, their profits are not always dependent on both of them. Consider $\{a_1, a_2\} \in \mathbb{K}_3$. Since the optimal prices for either I or E are zero, profits are independent of a_1 and a_2, i.e. they are constants. It is easy to verify that these are given by

$$\Pi_I^3 = -3c \qquad \text{and} \qquad \Pi_E^3 = -c$$

for $\{a_1, a_2\} \in \mathbb{K}_3$. For $\{a_1, a_2\} \in \mathbb{K}_2$, profits are given by

$$\begin{aligned}
\Pi_I^2(a_1) &= (p_I^D(a_1,0) - 2c)q_I\big(p_I^D(a_1,0),0\big) - cq_E\big(p_I^D(a_1,0),0\big) + a_1 p_I^D(a_1,0) \\
\Pi_E^2(a_1) &= -cq_E\big(p_I^D(a_1,0),0\big) - a_1 p_I^D(a_1,0).
\end{aligned}$$

It is important to realize that both I and E's profit turn out to be independent of a_2, hence I perceives his profit as being constant in stage 1.

Consider $\{a_1, a_2\} \in \mathbb{K}_4$. The corresponding price equilibrium alters perceived profits in the first stage to

$$\begin{aligned}
\Pi_I^4(a_2) &= -2cq_I\big(0, p_E^D(a_2,0)\big) - cq_E\big(0, p_E^D(a_2,0)\big) + a_2 p_E^D(a_2,0) \\
\Pi_E^4(a_2) &= (p_E^D(a_2,0) - c)q_E\big(0, p_E^D(a_2,0)\big) - a_2 p_E^D(a_2,0).
\end{aligned}$$

Because both profits are independent of a_1, E's choice on a_1 is irrelevant. Finally for $\{a_1, a_2\} \in \mathbb{K}_1$, profits are both dependent on a_1 and a_2.

$$\begin{aligned}
\Pi_I^1(a_1,a_2) &= (p_I^*(a_1,a_2) - 2c)q_I\big(p_I^*(a_1,a_2), p_E^*(a_1,a_2)\big) - cq_E\big(p_I^*(a_1,a_2), p_E^*(a_1,a_2)\big) + \\
&\quad + a_1 p_I^*(a_1,a_2) + a_2 p_E^*(a_1,a_2) \\
\Pi_E^1(a_1,a_2) &= (p_E^*(a_1,a_2) - c)q_E\big(p_I^*(a_1,a_2), p_E^*(a_1,a_2)\big) - a_1 p_I^*(a_1,a_2) - a_2 p_E^*(a_1,a_2).
\end{aligned}$$

To sum up, I perceives his profits to be constant for $\{a_1, a_2\} \in \mathbb{K}_3 \cup \mathbb{K}_4$. E's profits are constant for $\{a_1, a_2\} \in \mathbb{K}_3 \cup \mathbb{K}_2$. Notice that by choosing a_2, I is able to move from $\mathbb{K}_3$ to $\mathbb{K}_4$ and from $\mathbb{K}_2$ to $\mathbb{K}_1$. E on the other hand picks a_1, hence it could move from $\mathbb{K}_3$ to $\mathbb{K}_2$ and from $\mathbb{K}_4$ to $\mathbb{K}_1$.

Consider firm I's choice of a_2 and fix $a_1 < \alpha_1^4$. That means that at the margin, I chooses between $\mathbb{K}_3$ and $\mathbb{K}_4$. His profits are given by Π_I^3 and $\Pi_I^4(a_2)$, with $\Pi_I^4(\alpha_2^4) = \Pi_I^3$. Hence if $\frac{\partial \Pi_I^4}{\partial a_2}\big|_{a_2=\alpha_2^4} < 0$, I always prefers $\mathbb{K}_4$ to $\mathbb{K}_3$. It is easy to verify that $\frac{\partial \Pi_I^4}{\partial a_2} = c\sigma + \frac{1}{2} - a_2$, which is certainly negative for $a_2 = \alpha_2^4$. Hence I's optimal choice of a_2 is never in $\mathbb{K}_4$.

Now let us fix $a_1 \geq \alpha_1^4$. By choosing a_2, I chooses between $\mathbb{K}_2$ and $\mathbb{K}_1$. Notice that for $a_2 = 1 + c(1+\sigma) + \frac{\sigma}{2}\big(1 - c\sigma + a_1\big)$, $\Pi_I^1(a_1, a_2) = \Pi_I^2(a_1)$. If $\frac{\partial \Pi_I^4}{\partial a_2} < 0$ for $a_1 \geq \alpha_1^4$ and $a_2 = 1 + c(1+\sigma) + \frac{\sigma}{2}\big(1 - c\sigma + a_1\big)$, we can conclude that I prefers being in $\mathbb{K}_1$ rather than in $\mathbb{K}_2$. It is straightforward to verify that

$$\frac{\partial \Pi_I^4}{\partial a_2} < 0 \qquad \Leftrightarrow \qquad a_2 > c + \frac{(c\sigma^2 - 4)(2 - \sigma^2) + \sigma^3(a_1 + 1)}{2(3\sigma^2 - 8)} = \phi_2(a_1)$$

It is easy to see that $\phi_2(a_1) < 1 + c(1+\sigma) + \frac{\sigma}{2}\big(1 - c\sigma + a_1\big)$ for $a_1 \geq \alpha_1^4$. Hence, $\frac{\partial \Pi_I^4}{\partial a_2} < 0$ is indeed satisfied for $a_1 \geq \alpha_1^4$ and $a_2 = 1 + c(1+\sigma) + \frac{\sigma}{2}\big(1 - c\sigma + a_1\big)$. Hence we can conclude that I prefers $\mathbb{K}_1$ to $\mathbb{K}_2$.

Firm E decides upon a_1. If we fix $a_2 > \alpha_2^4$, E chooses between $\mathbb{K}_3$ and $\mathbb{K}_2$ by picking a_1, with profits being Π_E^3 and $\Pi_E^2(a_1)$ respectively. Notice that $\Pi_E^2(\alpha_1^4) = \Pi_E^3$. Since $\frac{\partial \Pi_E^2}{\partial a_1} = -(\frac{1}{2} + a_1 + c)$, which is positive for $a_1 = \alpha_1^4$, E prefers being in $\mathbb{K}_2$ rather than $\mathbb{K}_3$.

Now fix $a_2 \leq \alpha_2^4$. Picking a_1 amounts to choosing between $\mathbb{K}_4$ and $\mathbb{K}_1$, with profits given by $\Pi_E^4(a_2)$ and $\Pi_E^1(a_1, a_2)$ respectively. For $a_1 = -1 - 2c - \frac{\sigma}{2}\big(1 - c + a_1\big)$, $\Pi_E^4(a_2) = \Pi_E^1(a_1, a_2)$. If E prefers $\mathbb{K}_1$ to $\mathbb{K}_4$, we need to have $\frac{\partial \Pi_E^1}{\partial a_1} > 0$ for $a_1 = -1 - 2c - \frac{\sigma}{2}\big(1 - c + a_1\big)$ and $a_2 \leq \alpha_2^4$. Simple algebra shows that

$$\frac{\partial \Pi_E^1}{\partial a_1} > 0 \qquad \Leftrightarrow \qquad a_1 < \frac{4(1+2c)(2-\sigma^2) + \sigma^3(a_2 + c - 1)}{2(2\sigma^2 - 8)} = \phi_1(a_2)$$

It is straightforward to see that $\phi_1(a_2) > -1 - 2c - \frac{\sigma}{2}\big(1 - c + a_1\big)$ for $a_2 \leq \alpha_2^4$.

Summing up the findings, we have the following 'preferences' of I and E:

I: $\mathbb{K}_4 \succ \mathbb{K}_3$ and $\mathbb{K}_1 \succ \mathbb{K}_2$

E: $\mathbb{K}_2 \succ \mathbb{K}_3$ and $\mathbb{K}_1 \succ \mathbb{K}_4$

Notice that these orderings imply that $\mathbb{K}_1$ is the only set that survives. That means that any equilibrium in a_1 and a_2 necessarily has to lie in $\mathbb{K}_1$. ■

Lemma A.3.4 suggests that for finding an equilibrium, we can restrict ourselves to looking at $\Pi_I^1(a_1, a_2)$ and $\Pi_E^1(a_1, a_2)$ only. Notice that the best response function for a_1 and a_2 are given by $a_1^* = \phi_1(a_2)$ and $a_2^* = \phi_2(a_1)$ respectively.

Proposition A.3.7. *The unique equilibrium in the first stage of the game is given by*

$$a_1^* = \frac{\sigma^3 c - \sigma^2 - 2\sigma^2 c - 2\sigma - 4\sigma c + 4 + 8c}{\sigma^2 + 4\sigma - 8} \tag{A.3.22}$$

$$a_2^* = \frac{-8\sigma c + 5\sigma^2 c - 4 + 2\sigma + \sigma^2}{\sigma^2 + 4\sigma - 8} \tag{A.3.23}$$

Proof. Notice that $\phi_1(a_2)$ and $\phi_2(a_1)$ are both linear functions in a_1 and a_2. Therefore, the system of equation $a_1 = \phi_1(a_2)$ and $a_2 = \phi_2(a_1)$ have a unique solution. Simple algebra shows that this is given by (A.3.22) and (A.3.23). Furthermore, it is easy to verify that $\alpha_1^4 \leq a_1^* \leq 0$ and $0 \leq a_2^* \leq \alpha_2^4$. That means that $\{a_1^*, a_2^*\} \in \mathbb{K}_1$. ■

This completes the proof of Proposition 3.1

□

Proof of Proposition 3.2. Substituting (A.3.22) and (A.3.23) into (3.7) and (3.8), we get the symmetric price

$$p^* = \frac{2(2c\sigma - 2c - 1)}{(\sigma^2 + 4\sigma - 8)}.$$

Because of (3.17),

$$p_a^I(c) - p^* = \frac{(\sigma^2 + 2\sigma - 4) + 2c(3\sigma^2 - 2\sigma - 4)}{(2-\sigma)(\sigma^2 + 4\sigma - 8)} \tag{A.3.24}$$

Note that for all $\sigma \in]0;1[$, the denominator is negative. Both terms in brackets in the numerator are negative for all $\sigma \in]0;1[$, which makes (A.3.24) over all positive. This proves the first statement of the proposition.

To prove that welfare is higher under the $\{a_1, a_2\}$-mechanism, consider total surplus given by

$$TS = U(q_I, q_E) + (\Pi_i + \Pi_E)$$

with $U(q_I, q_E)$ given in (**??**). Rewriting $U(q_I, q_E)$ in terms of prices, taking the derivative with respect to p_I and evaluating this at a symmetric equilibrium yields

$$\left.\frac{\partial TS}{\partial p_I}\right|_{p_I = p_E = p} = \frac{2c(1-\sigma^2) - p(1-\sigma^3) + \sigma^2}{1+\sigma}.$$

Note that total surplus is decreasing in p, hence because of the first part of proposition 3.2 the claim is proved.

□

Proof of Lemma 3.1. To proof lemma 3.1, we have to solve the maximization program (3.20). Notice that the indirect utility function is given by:

$$U(p_I, p_E) = \frac{\sigma}{2(1-\sigma}(p_I + p_E)^2 - \frac{(1-p_I)^2}{2(1-\sigma)} - \frac{(1-p_E)^2}{2(1-\sigma)} + \frac{\sigma}{1-\sigma}(p_I + \sigma p_I p_E + p_E) \tag{A.3.25}$$

$$\mathfrak{L} = U(p_I, p_E) + l(\Pi_I + \Pi_2) \tag{A.3.26}$$

Differentiating w.r.t. p_I, p_E and l and dividing $\frac{\partial L}{\partial p_I} = 0$ by $\frac{\partial L}{\partial p_E} = 0$ eliminates l. The optimal Ramsey prices have to solve the two equations

$$\frac{q_I(p_I,p_E)}{q_E(p_I,p_E)} = \frac{q_I(p_I,p_E)+(p_I-2c)\frac{\partial q_I}{\partial p_I}+(p_E-2c)\frac{\partial q_E}{\partial p_I}}{q_E(p_I,p_E)+(p_I-2c)\frac{\partial q_I}{\partial p_E}+(p_E-2c)\frac{\partial q_E}{\partial p_E}} \tag{A.3.27}$$

$$\Pi_I+\Pi_E = 0 \tag{A.3.28}$$

where the l.h.s. of (A.3.27) is due to Roy's identity. Note that these are two quadratic formulae. Solving (A.3.28) for p_I, gives two solutions. We take the smaller of the two[25], which is given by

$$p_I^R(p_E)=\frac{1}{2}+c+\sigma(p_E-c)-\frac{D_1}{2} \tag{A.3.29}$$

where

$$\mathcal{D}_1=(2\sigma(p_E-c)+1)^2-4(f1+f2)-4p_E(p_E-2c-1)-4c(3-c(1-2\sigma)) \tag{A.3.30}$$

Notice that $p_I^R(p_E)\in\mathbb{R}$ iff $\mathcal{D}_1\geq 0$. This is true as long as

$$p_E\in\left[\frac{1}{2(1-\sigma)}+c-\frac{\sqrt{\mathcal{D}_2}}{2(1-\sigma^2)};\frac{1}{2(1-\sigma)}+c+\frac{\sqrt{\mathcal{D}_2}}{2(1-\sigma^2)}\right]$$

with

$$\mathcal{D}_2=2(1+\sigma)\big((1-2c(1-\sigma))^2-2(f1+f2)(1-\sigma)\big)$$

Substituting $p_I^R(p_E)$ in (A.3.27) and solving for p_E yields two solutions, The smaller one is the optimal symmetric Ramsey price given by:

$$p_R^*=\frac{1-2c(1-\sigma)-\sqrt{\mathcal{D}_3}}{2(1-\sigma)} \tag{A.3.31}$$

where

$$\mathcal{D}_3=\frac{\mathcal{D}_2}{2(1+\sigma)}$$

Hence, whenever $\mathcal{D}_2\geq 0$ there exists a symmetric Ramsey price given by p_R^* Note that $\mathcal{D}_2\geq 0$ implies

$$\frac{(1-2c(1-\sigma))^2}{1-\sigma}\geq 2(f_I+f_E)$$

which proves the result.

□

[25] Note that the Ramsey price has to be smaller than the multiproduct monopolist solution.

Proof of Proposition 3.3. Consider the function difference

$$\Theta(c,\sigma,F) = p_I^* - p_R^* = \frac{\sqrt{\mathcal{D}_3}}{2(1-\sigma)} - \frac{(4-\sigma^2)(1-2c(1-\sigma))}{(8-\sigma(4+\sigma))(1-\sigma)}$$

The proposition is proved by differentiation of the expression with respect to the three parameters, evaluated at $\Theta(c,\sigma,F)=0$. Note that the latter condition implies

$$\sqrt{\mathcal{D}_3} = \frac{2(4-\sigma^2)(1-2c(1-\sigma))}{8-\sigma(4+\sigma)} \tag{A.3.32}$$

Define

$$\begin{aligned}\beta_1 &= \frac{\sqrt{\mathcal{D}_3}}{2(1-\sigma)}\\ \beta_2 &= -\frac{(4-\sigma^2)(1-2c(1-\sigma))}{(8-\sigma(4+\sigma))(1-\sigma)}\end{aligned}$$

hence $\Theta(c,\sigma,F) = \beta_1+\beta_2$

i. **Differentiation w.r.t. σ:** Note that

$$\frac{\partial\beta_1}{\partial\sigma} = \frac{F+2c(1-2c(1-\sigma))}{2(1-\sigma)\sqrt{\mathcal{D}_3}} \geq 0$$

and

$$\begin{aligned}\frac{\partial\beta_2}{\partial\sigma} &= \overbrace{\frac{1}{2(8-\sigma(4+\sigma))(1-\sigma)}}^{>0}\Bigg(\overbrace{2\sigma(1+2c(1-\sigma))}^{>0} + \overbrace{2c(4-\sigma^2)}^{>0} + \\ &+ \underbrace{(4-\sigma^2)(1+2c(1-\sigma))}_{>0}\underbrace{\frac{(2\sigma+4)(1-\sigma)+(8-\sigma(4+\sigma))}{(8-\sigma(4+\sigma))(1-\sigma)}}_{>0}\Bigg) > 0\end{aligned}$$

for $\sigma \in [0;1]$ and $c \in [0;1/2]$. Hence, case (i.) of proposition 3.3 is proved.

ii. **Differentiation w.r.t. c:** Because

$$\frac{\partial\beta_1}{\partial c} = -\frac{1-2c(1-\sigma)}{\sqrt{\mathcal{D}_3}} \leq 0$$

and

$$\frac{\partial\beta_2}{\partial\sigma} = -\frac{4-\sigma^2}{8-\sigma(4+\sigma)} < 0$$

for $\sigma \in [0;1]$ and $c \in [0;1/2]$, $c > c^* \Rightarrow \Theta(c,\sigma,F) < 0$.

iii. **Differentiation w.r.t. F:** It is easy to see that

$$\frac{\partial \beta_1}{\partial F} = -\frac{1}{\sqrt{\mathcal{D}_3}} < 0$$

and

$$\frac{\partial \beta_2}{\partial \sigma} = 0$$

for $\sigma \in [0;1]$ and $c \in [0;1/2]$. Hence case (iii.) of proposition 3.3 is proved.

□

Proof of Proposition 3.4. The proof of proposition 4.3 follows immediately from the proof of proposition 3.2.

□

Chapter 4

Utility vs. Fixed Fee Competition and Interconnection Pricing

4.1 Introduction

Deregulation and technological progress have changed the telecommunications industry tremendously over the past two decades. Due to innovation in data transmission new players such as cable television companies are able to offer telecommunications services using their own network. As new services such as Voice over IP, video on demand etc. were technologically feasible, pricing schemes became much more involved and firms came up with more and more ornate tariff structures.

Up to this date, the regulation of interconnection charges remains the single most important policy question regulatory authorities have to answer in network industries. Starting with the pioneering work of Laffont et al. (1998a,b) and Armstrong (1998), interconnection pricing[1] has been identified to be a potential threat to competition. In particular, Gans and King (2001) suggest that with two-part tariffs and network-based price discrimination, access charges below marginal cost are used to soften competition, increase firms' profits and decrease overall welfare. In that case using more involved retail pricing schemes results in bad overall outcomes.

[1]Interconnection price or access charge is the payment from one network to another, in order to compensate the originating or terminating network for delivering the call to the receiver.

In this paper we employ a version of a two-part tariff to a model of two-way network interconnection introduced by Laffont, Rey, and Tirole (1998b) and Gans and King (2001). Two networks are horizontally differentiated and are able to price discriminate between on-net and off-net calls. However instead of charging a fixed fee to consumers, firms offer a net utility level ex-ante. Once market shares and overall call volumes[2] are realized, the fixed fee is computed ex post by means of a predetermined formula. This uses the announced utility levels, usage prices and market shares and is nothing but the inverse of the net surplus function which simply selects a price to equate realized and offered indirect utilities. Thus, information on the demand and indirect utility function is necessary to implement such a formula.

The paper shows that this retail price mechanism evaporates any collusive tendencies of the access charge. In equilibrium, firms price access exactly at cost and welfare is increased as compared to the "traditional" two-part tariff. Hence overall fixed fees and hence net-utility levels differ for both pricing regimes. We show that this difference is reminiscent of differences in Cournot and Bertrand prices and quanities in differentiated products markets. Singh and Vives (1984) and Cheng (1985) show that by committing to quantity competition, duopoly mark-ups are lager due to a decreased elasticity of demand as compared to Bertrand pricing. The same happens in the present setup. Committing to subscription fee competition allows firms to decrease the elasticity of subscription demand by lowering the access charge. Hence in equilibrium, the access price and per-unit off-net prices are distorted with subscription fee competition and welfare is lower than with utility based competition.

Furthermore we use a graphical argument in line with Cheng (1985) to show that subscription fee contracts dominate competition in utility space.

The paper is organized as follows. In section 4.2, we introduce the model and briefly review the results obtained by Laffont, Rey, and Tirole (1998b) and Gans and King (2001). Section 4.3 derives the subgame perfect equilibrium outcome with utility competition. In section 4.4 we compare our results to the literature and derive an endogenous contract. Section 4.5 concludes and discusses the relevance of the approach.

[2] It is straightforward to apply our analysis to other network industries such as utility, railway or postal industries. However for the remainder of the paper, we have the telecommunications industry in mind.

4.2 The Model

We use the framework of Laffont, Rey, and Tirole (1998a) thus allow destination based price discrimination and non-linear tariffs.

□ **Cost structure:** There are two networks 1 and 2, located on each end of a unit line. Both firms incur the same fixed cost f of providing service to a consumer. Marginal cost of providing a call consists of the cost of physically transporting data (c_0) and the cost of switching/routing the call (c_1). c_0 is incurred twice, since the call is transported from the caller to the switch and from thereon to the receiver, hence marginal cost of providing a call amount to $c = 2c_0 + c_1$.

□ **Demand structure:** In order to be as general as possible, we do not assume a specific model of demand formation. However, we restrict our attention to settings where markets are covered. Let α_i denote the market share of (demand for) network i, then $\alpha_j = 1 - \alpha_i$. Empirically this assumption seems to be reasonable for the case of telecommunication industries. Nearly every household already has a telephone, if not also a mobile set. This naturally implies that a decrease in the demand of network i is exactly offset by an increase in the demand of network j.

We assume that firms exhibit market power hence

$$\frac{\partial \alpha_i(w_i, w_j)}{\partial w_i} = \gamma(w_i, w_j) < 0$$

where w_i denotes the net surplus of being attached to network $i \in \{1, 2\}$. This derivative measures the mark up a firm is able to achieve because of its market power.

Set aside preferences for each specific networks, consumers are homogeneous with respect to calling patterns. This implies that there are no different types of users such as high or low users. In line with Laffont, Rey, and Tirole (1998b) the net-utility of making calls is given by $v(p) = \max_q u(q) - pq$, i.e. given a price for an outgoing call, a consumer picks the length of his call q in order to maximize his utility[3].

Furthermore, we assume balanced calling patterns, that is, every customer is equally likely to be called by everyone. It implies that the probability of a call terminating at either of the two networks is equal to its market share. Price discrimination between on-net and off-net calls and the behavioral assumptions imply net surplus functions for being

[3]We assume no income effects,hence $v'(p) = -q(p)$.

attached to firm 1 and 2 given by

$$w_1 = \alpha v(p_1) + (1-\alpha)v(p_{12}) - F_1 \tag{4.1}$$

$$w_2 = (1-\alpha)v(p_2) + \alpha v(p_{21}) - F_2 \tag{4.2}$$

where F_1 (F_2) denotes the subscription fee to be paid by consumers to firm 1 (2). The market share of firm 1 is denoted by α and, because markets are covered, firm 2 is left with a market share of $1-\alpha$.

□ **Access pricing:** Like most of the literature on two-way access pricing, we assume linear per-unit prices a. These are paid by the originating network to the terminating network[4]. Another assumption which is often suggested by regulatory authorities is the reciprocity of access charge. This assures that both networks pay the same price for transporting the same amount of traffic.

□ **Retail market competition:** The paper proposes a particular retail price mechanism. As opposed to a regular two-part tariff, where firms charge a subscription fee and per-unit-prices, we require firm i to announce a utility level w_i which they want to deliver to consumers and per-unit prices. Once these are announced and market shares are realized, subscription fees are determined ex post. These are computed according to the formula of the inverse of the net surplus function which is given by

$$F_i = \alpha_i v(p_i) + \alpha_j v(p_{ij}) - w_i. \tag{4.3}$$

Equations (4.1), (4.2) and (4.3) show the dichotomy between w_i and F_i. The equations constitute a demand framework similar to the traditional price/quantity relationship.

We allow firms to price discriminate between on-net (p_i) and off-net (p_{ij}) calls. The profit function of firm i is given by

$$\begin{aligned}\pi_i &= (F_i - f)\alpha_i + \alpha_i^2(p_i - c)q(p_i) + \alpha_i\alpha_j(p_{ij} - c)q(p_{ij}) \\ &\quad + \alpha_i\alpha_j mc\big(q(p_{ji}) - q(p_{ij})\big),\end{aligned} \tag{4.4}$$

where $m = \frac{a-c_0}{c}$ is defined as the access mark up relative to total marginal cost. The first term represents the profits due to subscription fees, the second and third terms represent profits from on-net and off-net calls, respectively. The last term denotes the access deficit.

[4]Observe that a could be negative. In that case the direction of the payment changes.

Whenever $m > 0$ ($m < 0$), it is positive (negative) if firm j transfers more traffic to network i than it receives.

□ **The timing of the game:** The strategic interaction is modeled by a two stage game. In stage 1, firms negotiate an interconnection price. Given this access charge, firms compete in stage two by offering the preferred level of overall utility and per-unit retail prices. Then consumers make subscription decisions after which the subscription fees are determined using (4.3).

4.3 Analysis

4.3.1 Competition in utility space

In this section we analyze the equilibrium outcome of the market game, given access charge a. That is, firms maximize their profits by choosing an optimal two part tariff consisting of per-unit retail prices and utility levels. It is instructive to rewrite (4.4) using (4.1) and (4.2), yielding

$$\begin{aligned}\pi_i &= (\alpha_i v(p_i) + \alpha_j v(p_{ij}) - w_i - f)\alpha_i + \alpha_i^2(p_i - c)q(p_i) \\ &\quad +\alpha_i\alpha_j(p_{ij} - c)q(p_{ij}) + \alpha_i\alpha_j mc\big(q(p_{ji}) - q(p_{ij})\big),\end{aligned} \tag{4.5}$$

Laffont, Rey, and Tirole (1998b) and Gans and King (2001) show that marginal on- and off-net prices equal (perceived) marginal cost of c and $(1+m)c$ respectively. A symmetric equilibrium is characterized by the first order conditions of both firms' profit functions with respect to w_i:

$$\begin{aligned}\frac{d\pi_i}{dw_i} &= \frac{\partial\alpha_i(w_i, w_j)}{\partial w_i}\Big(\alpha_i\big(mcq((1+m)c) + v(c)\big) + \alpha_j v((1+m)c) - w_1 - f\Big) \\ &\quad +\alpha_i\Big(\frac{\partial\alpha_i}{\partial w_i}\big(mcq((1+m)c) + v(c)\big) + \frac{\partial\alpha_j}{\partial w_i}v((1+m)c) - 1\Big)\end{aligned} \tag{4.6}$$

Applying market coverage ($\frac{\partial\alpha_i}{\partial w_i} = -\frac{\partial\alpha_j}{\partial w_i}$) and symmetry ($\alpha_i = \alpha_j = \frac{1}{2}$) we are left with

$$\frac{d\pi_i}{dw_i} = \frac{\partial\alpha_i}{\partial w_i}(v(c) - w_i - f) - \frac{1}{2} \tag{4.7}$$

The results can be summarized by Proposition 4.1:

Proposition 4.1. *Suppose two networks A and B offer a tariff which consists of two prices for on-net and off-net traffic (p_i and p_{ij} respectively) and an overall utility level w_i. Additionally an ex post payment F_i from consumers to firm i is determined by (4.3). If an equilibrium is characterized by the first-order conditions, this will be given by*

i. per-unit prices that are determined by

$$\begin{aligned} p_i^* &= c, \\ p_{ij}^* &= (1+m)c. \end{aligned}$$

ii. an overall utility level given by $w^* = v(c) - f - \frac{1}{2\gamma(w^*,w^*)}$

iii. the corresponding subscription fee for consumers is given by

$$F_i^* = \frac{1}{2}(v((1+m)c) - v(c)) + f + \frac{1}{2\gamma(w^*,w^*)}.$$

Proof: See appendix.

Part [i.] of Proposition 4.1 is a well known result which is common to most of the two-part tariff literature and it holds in our case as well. It simply states that usage prices are set equal to the perceived marginal cost of the operator. Part [ii.] characterizes the equilibrium utility level that is offered by the firms. In equilibrium, each firm offers utility independent of the access price — i.e. as if a consumer's calls are entirely on-net — net of the fixed cost incurred by the firm for connecting a single consumer and the mark-up that results from the sensitivity of market share with respect to net-utility levels. The assumption that γ, the sensitivity of own market share with respect to the offered net-utility level is independent of w_i in a symmetric equilibrium drives this result. It is easy to see in the appendix, that this allows us to solve for the symmetric equilibrium net utility. Part [iii.] of proposition 4.1 states the equilibrium subscription fee that is determined after the market shares are resolved. Its components are the fix cost of connection, the mark-up, and half of the loss (gain) of gross consumer surplus due to an access charge above (below) marginal cost.

There are two things to recognize in this proposition: firstly, agreed upon access charges do not enter the announced utility levels. Losses or gains in consumer surpluses due to above or below marginal cost pricing are fully incorporated by firms through adjusting of the subscription fee after the game has been played. It only enters the firms' offers via equilibrium off-net prices. Secondly, the firm is offering utility as if a consumer's calls are all on-net, i.e. priced at marginal cost c. Hence the distribution of market shares is not relevant for equilibrium net-utility levels. All asymmetries due to access charges and differences in market shares are accounted for ex post via the subscription fee which is derived from the inverse net surplus formula.

To compare our general result with the existing literature, we employ a Hotelling demand function that is frequently used in models of two-way interconnection.

If we adopt the Hotelling formulation applied in Laffont et al (1998b) and Gans and King (2001), the market share function is given by

$$\alpha_i(w_i, w_j) = \frac{1}{2} + \sigma(w_i - w_j)$$

and the derivative thereof reduces to

$$\frac{d\alpha_i}{dw_i} = \sigma \tag{4.8}$$

Hence we can state the following:

Corollary 4.1. *If market share result from a Hotelling model, then if either σ is sufficiently small or a is close to c_0, the equilibrium net-utility level is given by*

$$w^* = v(c) - f - \frac{1}{2\sigma} \tag{4.9}$$

where per-unit prices remain as in Proposition 4.1. The ex post subscription fee is given by

$$F_i^* = \frac{1}{2}(v((1+m)c) - v(c)) + f + \frac{1}{2\sigma}.$$

Proof: See appendix.

4.3.2 Determination of the access charge

Let us now turn to the first stage of the game, and derive the optimal access charge, i.e. joint profit maximizing level of access charge, given the retail market outcome. The interconnection price is negotiated cooperatively. The existing literature suggests that the access charge acts as a collusive device. We show that this is not the case if firms compete in net-utility levels and prices. In order to do so, let us rewrite (4.5) taking into account the symmetric equilibrium of the second stage given the access charge.

$$\pi_i = \frac{1}{4}\left[\frac{1}{\frac{\partial \alpha_i(w_i,w_j)}{\partial w_i}} - (v(c) - v((1+m)c)) + mcq((1+m)c)\right]. \tag{4.10}$$

Proposition 4.2. *Suppose that firms offer a tariff consisting of different off-net and on-net per-unit prices p_i^* and p_{ij}^* and an overall utility level w^*. Then, if market is covered and $\frac{\partial \alpha_i(w_i,w_j)}{\partial w_i}$ is independent of the access charge, the unique equilibrium access charge is chosen such that $a = c_0$, hence $m = 0$.*

Proof: See appendix.

Since the optimal access mark-up is zero in equilibrium, firms' perceived marginal cost equal their actual marginal cost and off-net prices are not distorted. From (4.10), we can identify three distinct effects on i's profit function of increasing m by an infinitesimal amount. First of all, it decreases the utility consumers derive from off-net calls and therefore decreases the subscription fee paid ex-post. This is exactly offset by a direct change of revenues due to the access price increase. At the same time, there is an indirect effect on access revenues because increasing the access charge decreases demand.

Since consumers' equilibrium utility level is independent of the access mark-up, firms fully incorporate changes of consumer surplus through changes in (ex-post) subscription fees. In the absence of income effects, these changes in surplus are offset by direct changes in access revenue due to an increase of the access mark-up. Thus, the only term we are left with is the change in profits due to the demand response to an increase in access charges. Marginal profits in this case can be set to zero by setting access price at cost of providing access. Hence, there are no incentives to distort the off-net cost prices, by taking away consumers' net-utility.

This is a very appealing result, since it suggests an efficient market outcome in the sense that firms price at marginal cost. It is resulting from the fact that firms are the only market agents who exhibit gains and losses from varying the access charge. This is a crucial difference to the models introduced by Laffont et al. (1998b) and Gans and King (2001). The next section provides an intuition for this difference.

4.4 Discussion

4.4.1 Differences in competition with utilities and flat fees

Since retail prices reflect (perceived) marginal cost, the difference in outcomes stems from the determination of subscription fees/net-utility levels. Using contracts based on subscription fees as in Laffont, Rey, and Tirole (1998b) and Gans and King (2001) we obtain

$$F_F^* = f + \frac{1}{2\sigma} - \Big(v(c) - v((1+m)c)\Big) \tag{4.11}$$

$$w_F^* = \frac{3}{2}v(c) - \frac{1}{2}v((1+m)c) - f - \frac{1}{2\sigma} \tag{4.12}$$

where the subscript F denotes the solutions in the case of competition in subscription fees. Taking differences between the results from proposition 4.1 we obtain

$$F^* - F_F^* = \frac{1}{2}(v(c) - v((1+m)c) = -(w^* - w_F^*). \tag{4.13}$$

This difference is zero for $m = 0$,[5] i.e. both results coincide for $m = 0$. For $m < 0$ — the equilibrium access markup obtained in Gans and King (2001) — net-utility supplied to consumers is higher in the case of utility based competition. The reverse result hold true for subscription fees. It is apparent from (4.12) that net-utility is not independent of the access charge. As we shall see, this has immediate welfare implications for both mechanisms.

If we want to compare both pricing regimes, we know that independent of whether firms compete in prices or utilities, marginal prices, i.e. p_i and p_{ij} are the same across both approaches. Thus, we can fix per-unit prices at their equilibrium values. Recall firm i's profit is given by

$$\pi_i = \alpha_i(F_i - f) + \alpha_i(\alpha_j)mcq((1+m)c) \tag{4.14}$$

where α_i denotes the market share of network i and

$$F_i = \alpha_i v(c) + \alpha_j v((1+m)c) - w_i.$$

Each firm's conjecture on the competitor's strategic behavior determines the duopoly outcome[6]. However, once firm j commits to a strategic variable and fixes his conjectures

[5] We are assuming identical access markups in both cases for this comparison.

[6] This point has been stressed in the literature on Bertrand vs. Cournot competition by Singh and Vives (1984) and Cheng (1985). See also Kamien and Schwartz (1983) and Grossman (1981).

about firm i's strategic variable, firm i is monopolist over his residual subscription demand. Therefore maximizing i's profit with respect to w_i and F_i is equivalent, once its competitors' conjectures are fixed.

Suppose firm i picks w_i, his first-order condition after simplifications, can be written as

$$\Big(2\alpha_i v(c) + (1-2\alpha_i)\big(v((1+m)c) + cmq((1+m)c)\big) - w_i - f\Big)\frac{d\alpha_i}{dw_i} - \alpha_i = 0. \tag{4.15}$$

Both competition in net-utilities and fixed fees ultimately result in a symmetric equilibrium, hence $\alpha_i = 1/2$. Solving (4.15) for w_i at $\alpha_i = 1/2$ yields

$$w_i = v(c) - f - \frac{1}{2\frac{d\alpha_i}{dw_i}}. \tag{4.16}$$

Firm j's conjectures on competition enter this expression via the total differential of α_i. If firm j commits to competing in fixed fees, it is equivalent to saying firm j commits to a utility rule $\bar{w}_j(F_j, w_i)$. Since, given F_j and the definition of α_i and α_j, w_j could be computed by solving

$$F_j = (\frac{1}{2} + \sigma(w_j - w_i))v(c) + (\frac{1}{2} - \sigma(w_j - w_i))v((1+m)c) - w_j,$$

yielding

$$\bar{w}_j(F_j, w_i) = \frac{1}{\Delta}\Big((\Delta - 1)w_i - F_j + \frac{1}{2}(v(c) + v((1+m)c))\Big). \tag{4.17}$$

with $\Delta = 1 - \sigma(v(c) - v((1+m)c))$. Therefore,

$$\begin{aligned}
\frac{d}{dw_i}\alpha_i(w_i, \bar{w}_j(F_j, w_i)) &= \sigma - \sigma\frac{\partial}{\partial w_i}\bar{w}_j(F_j, w_i) \\
&= \left(1 - \frac{\Delta - 1}{\Delta}\right)\sigma \\
&= \frac{\sigma}{1 - \sigma(v(c) - v((1+m)c))}.
\end{aligned} \tag{4.18}$$

Substituting (4.18) in (4.16) yields exactly (4.12).

Notice that (4.18) equals σ when $m = 0$, is smaller than σ if $m < 0$ and larger otherwise. Thus, if both firms commit to competing in subscription fees, they are able to decrease the elasticity of subscription demand by choosing $m < 0$. Hence, at the optimal access charge, characterized in Gans and King (2001), each firm faces a less elastic residual subscription demand. By decreasing the access charge, firms reduce the sensitivity of the subscription demand to utility levels. In doing so, they offer a higher value for off-net calls, however this increase in surplus is more than off-set by an increase in the subscription fee. If consumers' net-utility depends on the access charge, firms have an instrument for extracting surplus from the consumers.

4.4.2 Welfare Analysis

Overall welfare effects are, at first glance, ambiguous. Below cost pricing of off-net calls leads to losses and consumption of on-and-off-net calls are unambiguously distorted at the level of access charges characterized in Gans and King (2001). Hence with firms picking subscription fees, welfare is not at its optimal level.

Suppose a regulator wants to implement a benevolent social planner's choice and regulate market outcomes in order to maximize total surplus. In our analysis the benevolent dictator simply picks an access charge in the first stage and competition takes place in the second stage. Therefore there are two different situations, one with utility based and one with fixed fee competition in stage two. Hence, we can make use of the results of the preceding analysis. Consumer surplus is given by

$$CS = \alpha_i w_i + \alpha_j w_j - T$$

where T denotes the average disutility of not being subscribed to his preferred network with $T = t(\alpha^2 + (1-\alpha)^2)/2$. Total surplus is then given by

$$W = CS + \pi_i + \pi_j \tag{4.19}$$

Let us first look at the case of utility based competition. Given the equilibrium of the retail market game, we have

$$\begin{aligned} W_{util} &= CS + \pi_i + \pi_j \\ &= w^* - \frac{1}{4\sigma} + \frac{1}{2\sigma}\big[1 - \sigma(v(c) - v((1+m)c))\big] + \frac{1}{2}mcq((1+m)c) \\ &= \frac{1}{2}\Big[v(c) + v(1+m)c) + mcq((1+m)c)\Big] - f - \frac{1}{4\sigma}. \end{aligned} \tag{4.20}$$

Notice that we already made use of the fact that the equilibrium is symmetric. w^* is given by Proposition 4.1. The task for the social planner is to pick the access charge in order to maximize (4.20). Notice, however, that this is equivalent to the firms problem if they choose the profit maximizing access charge, since neither w^* nor $\frac{1}{4\sigma}$ is a function of the access charge. Therefore we can state the following proposition

Proposition 4.3. *Suppose firms are offering tariffs consisting of an overall utility level and per-unit charges. Then the profit maximizing access charge chosen by the firms is also socially efficient.*

Suppose that firms commit to competing in subscription fees. Taking into account (4.5), (4.11), (4.12) and the symmetry of the equilibrium, we can rewrite (4.3) as

$$\begin{aligned} W_{fix} &= CS + \pi_i + \pi_j \\ &= \frac{3}{2}v(c) - \frac{1}{2}v((1+m)c) - f - \frac{1}{2\sigma} - \frac{1}{4\sigma} + \\ &\quad \frac{1}{2\sigma} - \frac{1}{2}\left[2\Big((v(c) - v((1+m)c)\Big) - mcq((1+m)c)\right] \\ &= \frac{1}{2}\left[v(c) + v(1+m)c) + mcq((1+m)c)\right] - f - \frac{1}{4\sigma}. \end{aligned} \tag{4.21}$$

Observe that (4.20) and (4.21) are equal for the same m, hence their optimum is identical, namely $m = 0$. For any value of m social welfare is lower than at $m = 0$. However, Gans and King (2001) show that firms choose $m < 0$ in equilibrium if contracts are based on subscription fees. Thus, if firms compete in utilities the access charge is set at marginal cost, leaving per-unit prices undistorted and firms with Hotelling mark-ups.

Let us compare a firm's decision problem with both utility and subscription fee based competition. Differentiating a consumer's equilibrium net-utility of subscribing to network i with respect to m yields

$$\frac{\partial w^*}{\partial m} = \frac{1}{2}\frac{\partial v((1+m)c)}{\partial m} - \frac{\partial F^*}{\partial m}$$

With utility based competition, the l.h.s. is zero because of w^* in proposition 4.1. Hence

$$\frac{1}{2}\frac{\partial v((1+m)c)}{\partial m} = \frac{\partial F^*}{\partial m}.$$

Hence, changing m changes equilibrium profits as discussed in section 4.3:

$$\begin{aligned}\frac{\partial \pi^*}{\partial m} &= \frac{\partial}{\partial m}\left(\frac{1}{2}(F^* - f) + \frac{1}{4}mcq((1+m)c)\right) \\ &= \frac{1}{2}\frac{\partial F^*}{\partial m} + \frac{1}{4}cq((1+m)c) + \frac{1}{4}mc\frac{\partial q((1+m)c)}{\partial m} \\ &= \frac{1}{4}\frac{\partial v((1+m)c)}{\partial m} + \frac{1}{4}cq((1+m)c) + \frac{1}{4}mc\frac{\partial q((1+m)c)}{\partial m}\end{aligned}$$

Note that the first two terms of the last equality cancel out because of Shepard's Lemma. There is no distortion because the change in gross consumer surplus due to a change in access charge is fully absorbed by the change in the ex-post subscription fee.

Whenever firms compete in subscription fees the change in net consumer surplus due to a change in m is given by

$$\frac{\partial w_F^*}{\partial m} = \frac{1}{2}\frac{\partial v((1+m)c)}{\partial m} - \frac{\partial F_F^*}{\partial m}.$$

Taking into account the definition of w_F^* it is easy to see that this results in

$$\frac{\partial F_F^*}{\partial m} = \frac{1}{2}\frac{\partial v((1+m)c)}{\partial m} + \frac{1}{2}\frac{\partial v((1+m)c)}{\partial m}.$$

Hence the change in the profit due to a change in m is given by

$$\begin{aligned}\frac{\partial \pi_F^*}{\partial m} &= \frac{1}{2}\frac{\partial F^*}{\partial m} + \frac{1}{4}cq((1+m)c) + \frac{1}{4}mc\frac{\partial q((1+m)c)}{\partial m} \\ &= \frac{1}{2}\frac{\partial v((1+m)c)}{\partial m} + \frac{1}{4}cq((1+m)c) + \frac{1}{4}mc\frac{\partial q((1+m)c)}{\partial m}.\end{aligned}$$

The first two terms on the r.h.s. are equal to $-\frac{1}{4}q((1+m)c)$. Hence at $m = 0$ the reduction of demand for calls is more than offset by the increase in subscription charge. Hence each firm has an incentive to distort the prices for off-net calls. Consumers gain from lower off-net prices, but overall net utility is decreased by an increase in subscription fees.

Summing up, we have seen that for both modes of competition, the welfare maximizing access charge is equal to marginal cost. However, in an unregulated environment the utility based mechanism is clearly superior since it implements undistorted per-unit prices. The result is due to the fact that equilibrium net-utility levels are independent of the access charge. Hence losses in consumer surplus due to a deviation from cost-based access pricing are accounted for by firms via an ex-post decrease in the subscription fee. In case of subscription fee competition consumers have to pay a flat payment upfront and adjust their level of utility accordingly.

4.4.3 Endogenous Contracts

The preceding sections show that market outcomes and hence welfare is subject to the firms' choice parameters. This is reminiscent of the duality of Bertrand and Cournot competition. Singh and Vives (1984) and Cheng (1985) analyzed this issue in detail. Using a linear demands framework, the former show that with differentiated products, price competition is more efficient than quantity competition. Hence Bertrand prices are lower and subsequently, welfare is higher.

Furthermore Singh and Vives (1984) show that if firms are able to commit to either price or quantity contracts in a first stage, using quantities is a dominant strategy for substitutable products. If products are complements, quantity contracts are strictly dominated.

Coming back to the subject of the present paper, suppose that before firms engage in competition, they can precommit to either a subscription fee or utility contract. The question is, which strategy is dominated, if any. In order to answer this, we adopt a framework similar to Singh and Vives (1984), i.e. the game evolves as follows:

1. Firms decide non-cooperatively and simultaneously which contract to offer, either a two-part tariff with fixed fee or net utility level.

2. The access charge is negotiated.

3. Firms engage in retail competition according to their choice in the previous stage. Both pick a per-unit price and their respective usage independent component.

The game results in a 2×2 payoff matrix depicted in figure 4.1. Payoffs π_i^{ww} (π_i^{FF}) denote firm i's payoff if both firms decide to compete in utilities (fixed fees), the results of which are derived in previous section.

Symmetric equilibrium profits are determined by four-tuples given by $\{w^*, p_i^*, p_{ij}^*, m = 0\}$ for the case of utility competition and $\{F_F^*, p_i^*, p_{ij}^*, m < 0\}$ for competition in fixed fees.

To compute equilibria and graphically discuss results, we use a specific utility function

	w_2	F_2
w_1	π_1^{ww} π_2^{ww}	π_1^{wF} π_2^{wF}
F_1	π_1^{Fw} π_2^{Fw}	π_1^{FF} π_2^{FF}

Figure 4.1: Two Player Game

analog to the literature[7] on two-way interconnection of the form

$$u(q) = \frac{q^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}} \qquad \eta > 1$$

which yields a constant elasticity demand $q = p^{-\eta}$, an indirect utility function $u(p) = \frac{\eta}{\eta-1}p^{1-\eta}$ and net-utility of $v(p) = \frac{1}{\eta-1}p^{1-\eta}$.

Given the functional assumptions, we can compute the equilibrium profits π_i^{ww} and π_i^{FF}. Using the results of propositions 4.1 and 4.2, computing profits yields

$$\pi_i^{ww} = \frac{1}{4\sigma},$$

the Hotelling profit.

To compute equilibrium profits when both firms commit to fixed fee competition, we have to derive the optimal m, given our functional assumptions. With F_F^* defined by (4.11) every optimal m has to fulfill the necessary condition

$$\frac{c\big((1+m)c\big)^{-\eta}}{4(1+m)}\Big[1+m(1+\eta)\Big] \;=\; 0.$$

It is easy to see that the unique maximum[8] to that equation is given by

$$m_F^* = -\frac{1}{1+\eta} \;<\; 0.$$

Symmetric equilibrium profits are given by

$$\pi_i^{FF} = \frac{1}{4\sigma} + \frac{c^{1-\eta}}{4(\eta-1)}\left[\left(\frac{1+\eta}{\eta}\right)^{\eta} - 2\right].$$

[7]For example, CED demands are used in Laffont, Rey, and Tirole (1998a) and Berger (2002).

[8]Note that the second order condition is given by $\frac{\eta(1+\eta)mc\big((1+m)c\big)^{-\eta}}{4(1+m)^2}$. Since $\eta > 1$, $1-\frac{1}{1+\eta} > 0$, hence the second order condition is fulfilled for $m \in]-1,0]$ and satisfies the sufficient condition for a maximum.

In equilibrium, $\pi^{FF} > \pi^{ww}$ with $m^* < 0$. To show that subscription fee contracts are dominant for firm 1, we have to show that $\pi_1^{ww} < \pi_1^{Fw}$ and $\pi_1^{wF} < \pi_1^{FF}$. If $\pi_2^{ww} < \pi_2^{wF}$ and $\pi_2^{Fw} < \pi_2^{FF}$ is also satisfied, firm 2's dominant strategy is also competing in subscription fees. Hence competition in subscription fees is a unique pure strategy Nash equilibrium.

To derive the equilibrium of the game in figure 4.1 we use a graphical argument analog to Cheng (1985). We start with several helpful definitions that allow us to plot subscription fee equilibria for given $m \in]-1, 0]$ in $\{w_2, w_1\}$-space as in figure 4.2.

Firm i's iso-subscription fee curve $F_i(w_1, w_2)$ is defined as the line along which firm i charges the same subscription fee for different utility level pairs (w_2, w_1). Using (4.3) and applying the implicit function theorem, its slope is given by

$$\left.\frac{dw_1}{dw_2}\right|_{F_1(w)} = \frac{\sigma(v((1+m)c) - v(c))}{1 + \sigma(v((1+m)c) - v(c))}. \tag{4.22}$$

For $m \in]-1, 0]$, $\sigma \in [0, 1]$ and $c \in [0, 1]$ this is non-negative. It indicates, by which rate firm 1's has to increase net-utility offered in order to keep his subscription fee constant, if firm 2 increases his net-utility level by one unit.

The intuition for this is straightforward. Suppose firm 2 increases w_2. That means that customers move from network 2 to 1. Because of balanced calling patterns, that increases the fraction of calls made off-net. For $m \leq 0$ $v((1+m)c) \geq v(c)$, i.e. any given off-net call delivers at least as much utility as an off-net call. Hence with more customers subscribing to network 2, net-utility of being attached to network 1 is non-decreasing with F_1 constant. For $m < 0$ firm 1's net-utility is actually increasing. Firm 1's iso-subscription fee curve is therefore upward sloping and has a slope of less than one in $\{w_2, w_1\}$-space. By the same argument, firm 2's iso-subscription fee curve is also upward sloping and steeper than its competitors in $\{w_2, w_1\}$-space.

Further more, a firm's iso-subscription fee curve which is closer to the axis represents a higher subscription fee.

If firms compete in net-utility levels their reaction functions are characterized by their respective first-order conditions (4.6). Using the implicit function theorem its slope is given by

$$\left.\frac{dw_1}{dw_2}\right|_{R_i^w} = \frac{1}{2}\left[\frac{1 + 2\sigma(v((1+m)c) - v(c)) + 2\sigma mcq((1+m)c)}{1 + \sigma(v((1+m)c) - v(c)) + \sigma mcq((1+m)c)}\right]$$

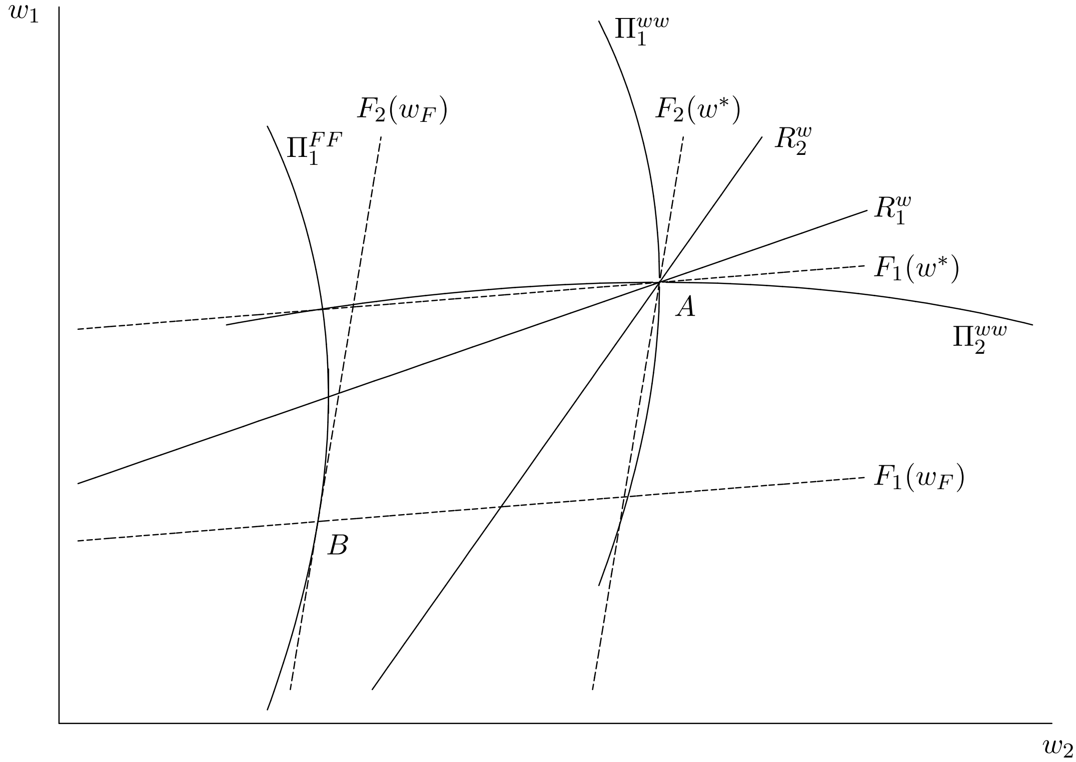

Figure 4.2: Subscription Fee Equilibrium in Utility Space

For $m \leq 0$ and c small enough, the slope of best response function is larger than that of the corresponding iso-subscription fee curve.

Define firm i's isoprofit curve $\Pi_i(b)$ as the curve in $\{w_2, w_1\}$-space that yields profit b, i.e. $\Pi_i(b) = \{w | w \geq 0, \pi_i(w) = b\}$. It's slope $\frac{dw_i}{dw_j}\big|_{\Pi_i}$ for $i \neq j$ is given by $-\frac{\partial \pi_I / \partial w_j}{\partial \pi_I / \partial w_i}$. Note that,

$$\frac{\partial \pi_i}{\partial w_i} = -\frac{\partial \pi_i}{\partial w_j} - \alpha \quad \Rightarrow \quad \frac{dw_i}{dw_j}\bigg|_{\Pi_i} = \frac{\frac{\partial \pi_i}{\partial w_i} + \alpha}{\frac{\partial \pi_i}{\partial w_i}} \tag{4.23}$$

for the case of Hotelling. When Π_i cuts i's best response curve, its slope is horizontal in $\{w_1, w_2\}$-space[9]. Because of (4.23) i's isoprofit curve is upward sloping for $w_1 < w_1^*(w_2)$ and downward sloping for $w_1 > w_1^*(w_2)$, where $w_1^*(w_2)$ denotes i's best response given w_2. In the absence of cornered markets $\frac{\partial \pi_i}{\partial w_j} < 0$, hence the closer i's isoprofit curve to the w_i-axis, the higher the profit.

In figure 4.2 point A depicts an equilibrium in net-utilities. Both firms' best response curves in net-utilities cross and the slopes of the isoprofit curves are horizontal. By def-

[9]Note that figure 4.2 is in $\{w_2, w_1\}$-space, hence the isoprofit's slope is vertical.

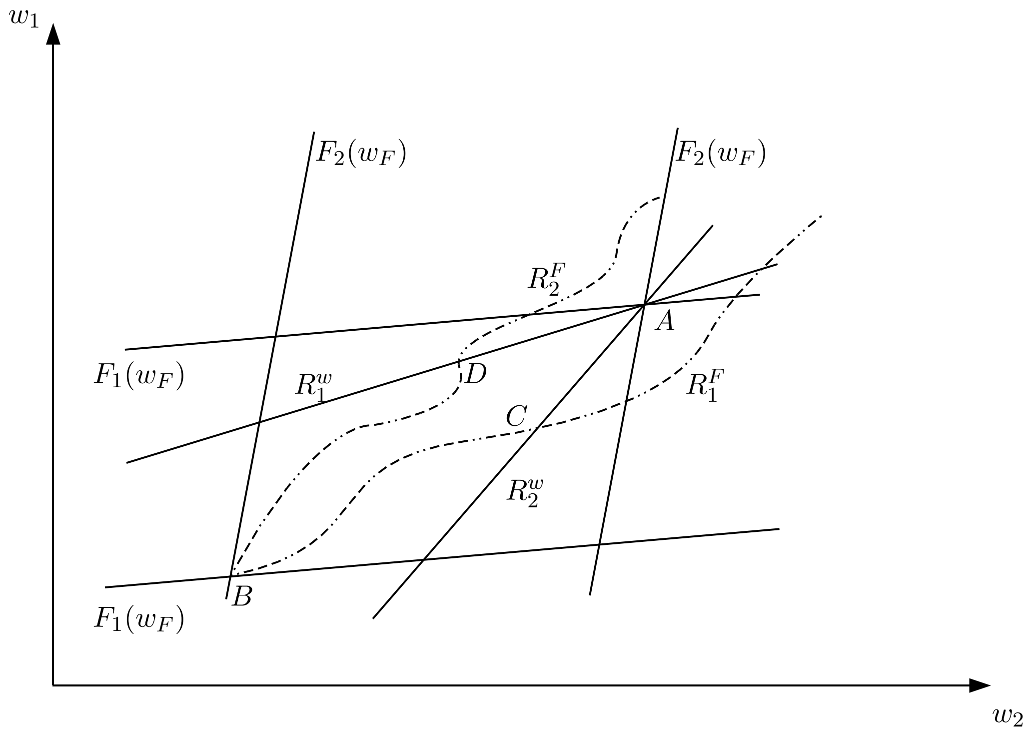

Figure 4.3: Utility Strategy vs. Subscription Fee Strategy

inition, $F_1(w^*)$ and $F_2(w^*)$ also intersect at A. These curves constitute all (w_1, w_2)-combinations where $F_1 = F_2 = F^*$ as defined in prop 4.1.

An equilibrium in subscription fees is constructed in point B of figure 4.2. The line $F_i(w_F)$ in figure 4.2 represents all (w_2, w_1)-combinations for which $F_i = F_F^*$, the optimal subscription fee in (4.11). Given $F_i = F_F^*$ the isoprofit curve that is tangent to $F_i(w_F)$ yields maximal profit.

The conclusions for a given mark-up m are similar to those of Cheng (1985) for the case of Cournot and Bertrand competition. Since the all iso-subscription fee curves have a positive slope and the characteristics of isoprofit curves, all points of tangency have to lie below R_1^w and to the left of R_2^w. It is easy to see that this implies both lower net-utilities and higher profits for at least one firm[10] in equilibrium.

Figure 4.3 derives the dominant strategy contract form for a given mark up m. Firm

[10]Cheng (1985) recognizes the fact that for asymmetrical demands the Cournot equilibrium can be such that one firm realizes less profits than in a Bertrand equilibrium. The same scenario is possible with subscription fee competition. Figure 4.2 however excludes this possibility.

i's choice of strategic variable induces firm j's conjectures, hence it defines firm j's reaction function.

In figure 4.3, firm i's best response function if net utilities are strategic variables is denoted by R_i^w. Firm i's best response to any subscription fee F_j R_i^F is given by the point of tangency of firm i's isoprofit curve and firm j's iso-subscription fee curve. From the discussion above, it is clear that these points need to be located below R_1^w for firm one and to the left of R_2^w for firm 2. A point of intersection of the subscription fee reaction curves defines a subscription fee equilibrium, i.e. point B in figure 4.3. If firm i commits to compete in subscription fees and firm j in net-utility levels, the equilibrium is given by the intersection of R_i^w and R_j^F. These are depicted by points C and D in figure 4.3.

Suppose that firm 1 chooses net-utility levels as a strategic variable, hence firm 2's relevant best response curve is R_2^w. If firm 2 also competes in net-utility levels, point A is the resulting equilibrium, because firm 1's relevant best response curve is R_1^w, both firms realizes $\pi_i^{ww} = \pi^{ww}$ for all $i = \{1, 2\}$. If firm 2 commits to net-utility levels as strategic variable, R_1^F is firm 1's reaction curve. Because of the shape of firm 1's isoprofit curves, the point of intersection of R_2^w and R_1^F has to be below A. In this case, firm 2 realizes higher profits, hence using net-utility levels as strategic variable is dominated for firm 2.

Suppose that firm 1 commits to subscription fees as strategic variable. This implies that firm 2's best response is given by R_2^F. If firm 2 also chooses a subscription fee as his strategic variable, the resulting equilibrium is certainly given by point B, yielding profits of π_2^{FF}.

If firm 2 picks net-utility levels instead, firm 1 faces R_1^w. The equilibrium $\{w_1, w_2\}$ combination is given by the point of tangency of $F_1(w)$ and firm 2's isoprofit curve that happens to be located on R_1^w. Because of the shape of isoprofit curves, this necessarily has to be above B. Hence choosing net-utility levels as strategic variable is dominated by choosing subscription fees.

The above results are derived by treating $m \in]-1, 0]$ as a constant parameter. However, m is also to the digression of the firms. Nevertheless it is easy to see, that choosing subscription fees is still a dominant strategy for either firm, if m is variable. Suppose there exists a $\hat{m} \in]-1,0]$ such that

$$\pi_i^{Fw}(\hat{m}) \geq \pi_i^{Fw}(m) \qquad \forall\, m \in]-1,0].$$

We already know that $\pi_i^{FF}(m) > \pi_i^{Fw}(m)$, $\forall\, m \in]-1,0]$ hence $\pi_i^{FF}(\hat{m}) > \pi_i^{Fw}(\hat{m})$. But since $\pi_i^{FF}(m^*) > \pi_i^{FF}(m)\ \forall\, m \in]-1,0]$, choosing subscription fees with $m = m^*$ is indeed the unique equilibrium of the game depicted in table 4.1.

4.5 Conclusion

This paper proposes a mechanism which vaporizes the collusive nature of the access charge in the presence of on and off-net price discrimination by using a special form of a nonlinear tariff. It also forms a link between the literature on network interconnection and the comparison of price and quantity competition. The three major findings of the paper are:

i. By committing to utilities instead of prices, the sensitivity of the residual subscription demand is altered as compared to the case of committing to subscription fees. This affects the equilibrium values of net-utility/subscription fees. Hence, in the presence of network effects, price and utility competition are not equivalent.[11]

ii. The equilibrium net-utility level offered to consumers is independent of the access charge when firms commit to utilities. Hence, the access charge chosen by profit maximizing firms and the socially optimal level coincide. This is not the case in models of network competition with price discrimination and two-part tariffs.

iii. Competing in subscription fees is dominant to competing in net-utility levels.

The paper suggests that the regulator should impose competition in net utilities instead of subscription fees. If they do so, there will be an equilibrium which immediately results in the socially optimal outcome. Even though these results are very intriguing, competition in utilities in its pure form is not used in practice. Although many reasons for its non-existence in reality can be brought forward, we only want to highlight two.

The first one is suggested by the paper itself. If firms had to decide between utility and subscription fee competition, the paper argues that the latter dominates the former.

[11] This has been realized by Armstrong (2002a) in the context of two-sided markets.

Hence, profit maximizing firms clearly refrain from competing in utilities. However, the analysis in Cheng (1985) and Singh and Vives (1984) suggests that quantity competition dominates price competition, but it would be ignorant to argue that price competition does not exist. The mode of competition is certainly not only subject to the firms interest. Pricing strategies and habits rather evolved over time. Hence the analysis of the paper suggests that it is in the firms' best interest to engage in fixed fee competition, but there are certainly other factors that influence that decision.

The second obstacle is the more obvious one — at least at first sight. Net utilities are a theoretical concept and hence, it is almost impossible to directly implement such a pricing scheme. Certainly, no telecommunication firm would advertise its network with the words: "Subscribe now and get 10 utilis a month!".

This problem is not unique to the present proposition. For example, although it is theoretically true that network usage has a marginal cost (other than causing congestion), we use proxies and techniques to recover estimates. We implicitly assume that these proxies draw a sufficiently accurate picture of the world and accept our inability to directly observe marginal cost.

In our model net-utility is essentially a function of the amount of people that one can reach via its network. In that case the number of calls dropped and network reach in particular can be used as proxies. These are measurable and verifiable. As a matter of fact reliability is at least a marketing instrument in the US. Verizon Wireless advertises its mobile network with the slogan, "The most reliable network".

Since pure contracts based on net-utility seem to be far fetched (at least for now), mixed contracts of some measure of utility and fixed fee pricing could be more realistic. A natural extension of the model are supply functions. These have been studied extensively for the case of price and quantity competition (Look at Grossman (1981) and Klemperer and Meyer (1989) for further references.) Further research would need to identify consequences of the use of supply functions on the equilibrium access charge.

4.A Appendix

Proof of Proposition 4.1. Differentiating (4.5) with respect to p_i and p_{ij} and taking into account that $v'(p) = -q(p)$ yields first-order conditions given by

$$\begin{aligned} \alpha_i^2(p_i - c)q'(p_i) &= 0, \\ \alpha_i\alpha_j(p_{ij} - (1+m)c)q'(p_{ij}) &= 0, \end{aligned}$$

implying the per unit retail price given in part [i.].

Differentiating (4.5) with respect to w_i, and substituting the retail prices from part [i.], and imposing symmetry, the first-order condition of firm i determining w_i can be written as

$$\gamma(w_i, w_j)v(c) - \gamma(w_i, w_j)w_i - \gamma(w_i, w_j)f - \frac{1}{2} = 0.$$

Solving this equation for w_i yields the expression given in part [ii.].

In order to compute the subscription fee, one has to plug in w^* into (4.3). solving for F_i leads to [iii.] in proposition 4.1. □

Proof Corollary 4.1. The net-utility level in (4.9) follows immediately by plugging in (4.8) in Proposition 4.1. The second order condition is given by

$$2\sigma(-mcq((1+m)c) + \sigma\Big(v(c) - v((1+m)c)\Big) - 1) < 0$$

which proofs the corollary. □

Proof of proposition 4.2. In order to proof that $m = 0$ is indeed optimal, we have to differentiate (4.10) with respect to m. Again, taking into account that $v'(p) = -q(p)$ gives the following first-order condition

$$\frac{1}{4}mc\frac{\partial q((1+m)c)}{\partial m} = 0$$

observing that $\partial q((1+m)c)/\partial m < 0$ and $c > 0$ we have $m = 0$ as the unique solution, which is stated in proposition 4.2.

Furthermore observe that the second derivative of (4.10) is given by

$$\frac{1}{4}c\frac{\partial q((1+m)c)}{\partial m} + \frac{1}{4}mc\frac{\partial^2 q((1+m)c)}{\partial m^2}.$$

This is always smaller then zero if

$$mc \lesseqgtr -\frac{\partial q((1+m)c)/\partial m}{\partial^2 q((1+m)c)/\partial m^2}$$

depending on whether $\partial^2 q((1+m)c)/\partial m^2 \gtreqless 0$. For $m = 0$ this is always true. $\square$

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Eidesstattliche Erklärung:

Ich erkläre hiermit an Eides Statt, dass ich meine Doktorarbeit "Pricing and Interconnection Agreements in Network Markets" selbständig und ohne fremde Hilfe angefertigt habe und dass ich alle von anderen Autoren wörtlich übernommenen Stellen, wie auch die sich an die Gedanken anderer Autoren eng anlehnenden Ausführungen meiner Arbeit, besonders gekennzeichnet und die Quellen nach den mir angegebenen Richtlinien zitiert habe.